ON OPTIMAL BEAM STEERING DIRECTIONS IN MILLIMETER WAVE SYSTEMS

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ABSTRACT

In order to increase system capacity and transmission rates, fifth-generation (5G) cellular and 802.11ad/ay wireless systems will use millimeter wave frequencies (24GHz-70GHz), which require large antenna arrays and narrow beamforming to mitigate the impact of substantial path-loss. While narrow beamforming maximizes array gain it also entails a significant increase in the number of beams needed to maintain the desired cell coverage. Consequently, there is a considerable increase in the complexity of beam management for initial user access and then tracking. This paper proposes a novel beam steering algorithm to maximize coverage given a prescribed size for the codebook of beam directions. The proposed approach leverages the observation that the problem of finding the optimal set of beam pointing angles can be mapped to the classical quantizer design problem in source coding, albeit with a somewhat unusual distortion measure. A variant of the Generalized Lloyd Algorithm is derived and employed to find the optimal codebook of beam placement angular directions. Numerical results show up to 2 dB gains in average power array factor, in comparison with the conventional uniformly spaced beam steering approach. These performance gains can be traded for codebook size reduction and a corresponding reduction in beam management complexity.

***Index Terms*—** Beamforming, Beam Steering, MIMO, Millimeter Wave, 3D Channel Model

1. INTRODUCTION

In response to the ever-increasing demand for higher data-rates, much attention has been directed at millimeter wave multiple-input multiple-output (MIMO) systems [1, 2], where a considerable increase in data-rates is expected by exploiting the large swaths of potentially usable bandwidth. However, extensive path-loss and signal attenuation is experienced at such high frequencies, which impose fundamental limits on cell size. Furthermore, recent studies have shown a number of additional challenges [3, 4]. In order to maintain an acceptable link quality, the communication devices employ large antenna arrays with narrow beamforming that enhance the array gain, and compensate for significant losses through the channel. Given a transmit linear array of length $N_{\mathtt{tx}}$, the increase in Effective Isotropic Radiated Power (EIRP) is proportional to $N_{\mathtt{tx}}$ [5]. However, the Half-Power Beam Width (HPBW) is in fact inversely proportional to $N_{\mathtt{tx}}$. The use of narrow beams naturally entails an increase in the number of beams in the codebook to reasonably cover a defined geographical space, and thereby a corresponding increase in the beam management complexity. For example, in 5G New Radio (NR) standards, the base station sweeps beams in its codebook both periodically (e.g., in the Synchronization Signal (SS) blocks) and as needed (e.g., in the Channel State Information Reference (CSIR) signal transmissions to user devices) in order to track users and maintain links [6]. The narrower the beams, the greater the sweep, measurement, and reporting complexity. Moreover, the latency due to this sweeping process ultimately renders the system less robust to dynamics. Note that 802.11ad/ay systems employ a similar beam sweep procedure to establish and maintain links [7].

Given a specified coverage area, codebook size (denoted as N_b), and beam width, the beam steering algorithm determines the average performance seen over all user positions in the defined geographical coverage space. The problem of finding the optimal beam steering angles can, in fact, be viewed as a quantizer design problem, where the two-dimensional angular space is quantized into N_b pointing angles. As you increase the number of pointing angles, the average link performance over the angular space increases, but the rate of beam updates increases as well, and the system becomes less robust to dynamics. This trade-off is analogous to the classical rate-distortion trade-off. As the quantizer design, or clustering problem, appears with various flavors in many diverse applications, solution methods have been developed in different disciplines. In the communications or information-theory literature, an early clustering method was suggested for scalar quantization, variants of which are known as the Lloyd algorithm [8] or the Max quantizer [9]. This method was later generalized to vector quantization, and to a large family of distortion measures [10], and the resulting algorithm is commonly referred to as the Generalized Lloyd Algorithm (GLA). In the pattern-recognition literature, similar algorithms have been introduced including the ISODATA [11] and the K-means [12] algorithms. All these iterative methods alternate between two complementary steps: optimization of the partition into clusters given the current codebook entries, and optimization of the codebook entries for their respective clusters. It is easy to show that this iterative procedure is monotone non-increasing in the distortion. Hence, convergence to a local minimum of the distortion is ensured. The objective of this paper is to find the optimal beam steering directions, i.e., design the optimal beamforming codebook. Specifically, this work develops a novel beam steering approach based on GLA.

The remainder of this paper is organized as follows: Section 2 defines the system model. Section 3 illustrates the beamforming setup considered. The novel beam steering algorithm is introduced in Section 4. Section 5 summarizes the numerical results on system performance compared to conventional beam steering techniques. Finally, conclusions are drawn in Section 6.

2. SYSTEM SETUP

The setups considered include an outdoor 5G cellular scenario at 28 GHz carrier frequency and an indoor wireless LAN scenario at 60 GHz carrier frequency. The outdoor base station (also referred to as

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gNB) or the indoor Access Point (AP) are equipped with a planar array consisting of N_{tx} antennas, while the User Equipment (UE) or the Station (STA) are equipped with a planar array consisting of N_{rx} antennas. Typical values of array sizes are (32×8), and (5×5) for gNB and AP, respectively. For UEs or STAs, the typical array sizes are (4×1) and (4×4), respectively. The channel considered between the transmitter side and receiver side follows the 3GPP Cluster Delay Line (CDL) channel model in [13]. For a given channel, let N_{c} be the number of detected clusters, and M_{r} the number of rays within a single cluster. Let $m \in \{1, 2, \ldots, M_{\mathrm{r}}\}$ denote the ray index, and $n \in \{1, 2, \ldots, N_{\mathrm{c}}\}$ the cluster index. Moreover, let the $(N_{\mathrm{rx}} \times N_{\mathrm{tx}})$ channel matrix be denoted by $\mathbf{H}_{n,m}(t)$, where t is the time index. Next, let $\mathbf{b}_{\mathrm{tx}}(\boldsymbol{\varphi})$ and $\mathbf{b}_{\mathrm{rx}}(\boldsymbol{\vartheta})$ denote the unit-norm phase-control $(N_{\mathrm{tx}} \times 1)$ transmit beamforming vector and the $(N_{\mathrm{rx}} \times 1)$ receive beamforming vector, respectively. Such that $\boldsymbol{\varphi}$, and $\boldsymbol{\vartheta}$ are the transmit and receive vectors of the beamforming phases. The system model, in this setting, is given by,

$$y(t,f_r) = (\mathbf{b}_{\mathrm{rx}}(\boldsymbol{\vartheta}))^{\mathrm{H}} \sum_{n=1}^{N_{\mathrm{c}}} \sum_{m=1}^{M_{\mathrm{r}}} \left\{ \left(\mathbf{H}_{n,m}(t) e^{-j2\pi f_r \tau_n(t)} \right) \mathbf{b}_{\mathrm{tx}}(\boldsymbol{\varphi}) x(t,f_r) \right\} + (\mathbf{b}_{\mathrm{rx}}(\boldsymbol{\vartheta}))^{\mathrm{H}}\, \mathbf{n}(t,f_r), \tag{1}$$

where f_r is the rth sub-carrier frequency, $\tau_n(t)$ is the nth cluster delay, $x(t, f_r)$ is the complex frequency domain transmit symbol with $\mathbb{E}\left[|x(t, f_r)|^2\right] = 1$, and $\mathbf{n}(t, f_r) \sim \mathcal{CN}(0, \sigma_n^2)$ is the complex AWGN vector. Additionally, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote transpose and conjugate-transpose operations, respectively. The noise power of the elements in $\mathbf{n}(t, f_r)$ is obtained as,

$$\sigma_n^2 = k_{\mathrm{B}} T B, \tag{2}$$

where k_{B} is the Boltzmann constant, T is the temperature and B is the transmission bandwidth. The perceived channel coefficients, upon applying the beamforming vectors, are given by

$$h_{n,m}(t,f_r) = (\mathbf{b}_{\mathrm{rx}}(\boldsymbol{\vartheta}))^{\mathrm{H}}\, \mathbf{H}_{n,m}(t) e^{-j2\pi f_r \tau_n(t)} \mathbf{b}_{\mathrm{tx}}(\boldsymbol{\varphi}). \tag{3}$$

which yields the aggregate channel transfer function, due to all clusters and rays,

$$h(t, f_r) = \sum_{n=1}^{N_{\mathrm{c}}} \sum_{m=1}^{M_{\mathrm{r}}} h_{n,m}(t, f_r). \tag{4}$$

The Signal-to-Noise Ratio (SNR) at the rth sub-carrier with frequency f_r is given by,

$$\gamma_r(t) = \frac{P_{\mathrm{tx}} G_{\mathrm{tx}} |h(t, f_r)|^2 G_{\mathrm{rx}}}{P_{\mathrm{L}}(t) F_n\, \sigma_n^2}, \tag{5}$$

where P_{tx} is the average transmit power, $P_{\mathrm{L}}(t)$ is the path-loss, and F_n is the receiver noise factor. The transmit and receive antenna elements' maximum gains are defined as G_{tx} and G_{rx}, respectively.

3. BEAMFORMING FRAMEWORK

Beamforming can be attained using either amplitude control, phase control, or both. For maximum power efficiency and maximum total transmit power, it is desirable to operate the power amplifier associated with each antenna as close to its saturation point as possible. Typically, to avoid non-linear effects, the operating point is selected to be a few dB below the saturation point (also called back off), to allow for some peak-to-average power margin. Amplitude-based beamforming is sub-optimal due to the drop seen in the EIRP when power amplifiers are either switched off or operating well below the

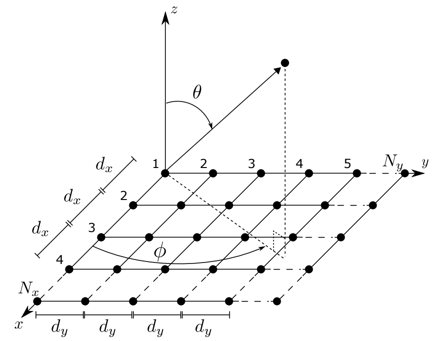

Fig. 1. Coordinate system of planar array with uniform spacing.

optimal efficiency point. Beam broadening achieved by switching off $(\ell - 1)N_{\mathrm{tx}}/\ell$ power amplifiers, with $\ell \in \left\{2^1, 2^2, \ldots, N_{\mathrm{tx}}\right\}$ is a special case of amplitude-based beamforming. For example, turning off half the power amplifiers results in 6 dB drop in EIRP at the steering direction. This is because the total transmit power decreases by 3 dB and the power array factor drops by 3 dB as well. Correspondingly, the beam width increases by a factor of two. Due to the severe loss in EIRP when using amplitude-based beam broadening, throughout the remainder of this paper, only phase-control based beamforming will be considered.

In the subsequent analysis, we will focus on transmit beamforming only, but similar analysis can be done for receive beamforming. Throughout this paper, we consider a planar antenna array with uniform spacing, i.e., $d_x = d_y = \frac{\lambda_c}{2}$. Note that results and conclusions drawn in this paper are extended to any antenna array configuration. Define the beam-space transformation as $\Omega_x = k d_x \sin(\theta)\cos(\phi) = \pi \sin(\theta)\cos(\phi)$, and $\Omega_y = k d_y \sin(\theta)\sin(\phi) = \pi\sin(\theta)\sin(\phi)$, where $k = \frac{2\pi}{\lambda_c}$ is the wave number. The planar array setup is depicted in Fig. 1. Hence, the transmit array factor simplifies to [14],

$$\mathbf{a}_{\mathrm{tx}}\left(\Omega, N\right) \triangleq \left[1\; e^{-j\Omega}\; \ldots\; e^{-j\Omega(N-1)}\right]^{\mathrm{T}}, \tag{6}$$

$$\mathbf{b}_{\mathrm{tx}}\left(\boldsymbol{\varphi}\right) \triangleq \mathbf{b}_{\mathrm{tx}}^{(x)}\left(\boldsymbol{\varphi}_x\right) \otimes \mathbf{b}_{\mathrm{tx}}^{(y)}\left(\boldsymbol{\varphi}_y\right), \tag{7}$$

$$A_{\mathrm{tx}}(\Omega_x, \Omega_y, \boldsymbol{\varphi}_x, \boldsymbol{\varphi}_y) = \left(\mathbf{a}_{\mathrm{tx}}\left(\Omega_x, N_x\right)^{\mathrm{H}} \mathbf{b}_{\mathrm{tx}}^{(x)}(\boldsymbol{\varphi}_x)\right) \times \left(\mathbf{a}_{\mathrm{tx}}\left(\Omega_y, N_y\right)^{\mathrm{H}} \mathbf{b}_{\mathrm{tx}}^{(y)}(\boldsymbol{\varphi}_y)\right), \tag{8}$$

where $\mathbf{b}_{\mathrm{tx}}^{(x)}(\boldsymbol{\varphi}_x)$ and $\mathbf{b}_{\mathrm{tx}}^{(y)}(\boldsymbol{\varphi}_y)$ are the beamforming vectors across x-dimension and y-dimension, respectively. The Kronecker product operation is denoted by $\otimes$. The normalized phase-control beamforming vectors can be defined as

$$\begin{aligned} \mathbf{b}_{\mathrm{tx}}^{(x)}(\boldsymbol{\varphi}_x) &= \frac{1}{\sqrt{N_x}} \left[e^{j\varphi_1^{(x)}}\; e^{j\varphi_2^{(x)}}\; \ldots\; e^{j\varphi_{N_x}^{(x)}}\right]^{\mathrm{T}}, \\ \mathbf{b}_{\mathrm{tx}}^{(y)}(\boldsymbol{\varphi}_y) &= \frac{1}{\sqrt{N_y}} \left[e^{j\varphi_1^{(y)}}\; e^{j\varphi_2^{(y)}}\; \ldots\; e^{j\varphi_{N_y}^{(y)}}\right]^{\mathrm{T}}. \end{aligned} \tag{9}$$

Assuming that only phase-control is allowed, the beam with the maximum absolute array factor at beam steering angles, $\omega_x = \pi\sin(\theta_0)\cos(\phi_0)$ and $\omega_y = \pi\sin(\theta_0)\sin(\phi_0)$, can be attained using the conventional Constant Phase Offset (CPO) beamforming technique [5, 14, 15], where ϕ_0 and θ_0 are the azimuth and elevation pointing angles. The beamforming vectors for the CPO technique are obtained as,

$$\begin{aligned} \mathbf{b}_{\mathrm{tx}}^{(x)}(\omega_x) &= \frac{1}{\sqrt{N_x}} \left[1\; e^{-j\omega_x}\; \ldots\; e^{-j\omega_x(N_x-1)}\right]^{\mathrm{T}}, \\ \mathbf{b}_{\mathrm{tx}}^{(y)}(\omega_y) &= \frac{1}{\sqrt{N_y}} \left[1\; e^{-j\omega_y}\; \ldots\; e^{-j\omega_y\left(N_y-1\right)}\right]^{\mathrm{T}}. \end{aligned} \tag{10}$$

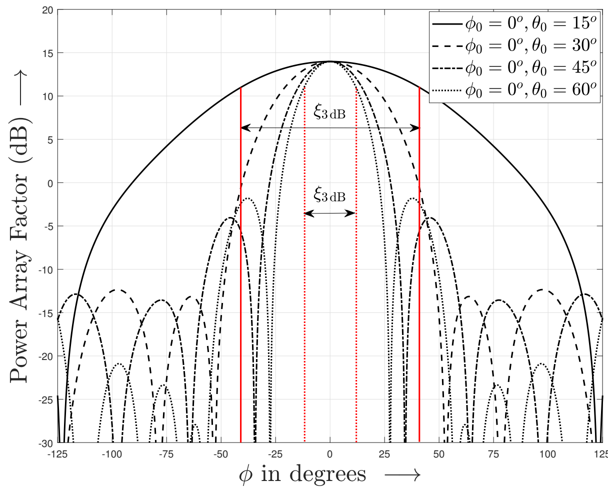

Fig. 2. The power array factor slice for 5×5 square planar array using CPO beamforming. For each steering direction, the slice is taken at $\theta = \theta_0$.

Therefore, the per-dimension absolute array factor can be written as [14],

$$\begin{aligned} &\left|A_{\text{tx}}^{(x)}([\phi\ \theta]^{\text{T}}, [\phi_0\ \theta_0]^{\text{T}})\right| = \\ &\frac{1}{\sqrt{N_x}} \left[\frac{\sin\left(\frac{N_x \pi}{2}\left(\cos(\phi)\sin(\theta) - \cos(\phi_0)\sin(\theta_0)\right)\right)}{\sin\left(\frac{\pi}{2}\left(\cos(\phi)\sin(\theta) - \cos(\phi_0)\sin(\theta_0)\right)\right)} \right], \\ &\left|A_{\text{tx}}^{(y)}([\phi\ \theta]^{\text{T}}, [\phi_0\ \theta_0]^{\text{T}})\right| = \\ &\frac{1}{\sqrt{N_y}} \left[\frac{\sin\left(\frac{N_y \pi}{2}\left(\sin(\phi)\sin(\theta) - \sin(\phi_0)\sin(\theta_0)\right)\right)}{\sin\left(\frac{\pi}{2}\left(\sin(\phi)\sin(\theta) - \sin(\phi_0)\sin(\theta_0)\right)\right)} \right]. \end{aligned} \tag{11}$$

Without loss of generality, the CPO beamforming technique will be assumed in the subsequent analysis. All results and conclusions are extendable to any beamforming method.

4. OPTIMAL BEAM STEERING

This section focuses on new codebook methods to approach beam steering optimality. The beamforming vectors are stored in codebook entries, such that each entry specifies an angular direction. Specifically, each codebook entry corresponds to an elevation angle and an azimuth angle. The simplest beam steering approach is to quantize the elevation and azimuth field-of-view *uniformly* into N_b pointing directions, similar to [16], where N_b is the number of beams (entries) in the codebook. It is important to note that the beam width is direction-dependent, i.e., different beam steering angles result in a wider or narrower beam width as depicted in Fig. 2, where $\xi_{3\text{dB}}$ is the 3-dB azimuth beam width in degrees. Furthermore, although the UEs or STAs are often assumed to be uniformly distributed across the horizontal plane [13], this nevertheless results in a non-uniform distribution of the users' angles ϕ_i and θ_i, where i is the user index. Clearly, uniform distribution of beam steering angles across the field-of-view is suboptimal.

We propose an iterative framework that converges to (at least locally) optimal performance. First, it is useful to recognize that the beam steering problem at hand is effectively equivalent to a generalized quantizer design problem (albeit with an unusual distortion measure). The space to be quantized is the 2-dimensional angular space, and the space boundaries are identified by the transmitter field-of-view. For example, if we consider an indoor wireless LAN settings, the field-of-view of the APs mounted on the ceiling is $\phi \in [-180^o, 180^o]$ and $\theta \in [0^o, 90^o]$, thus the angular space to be quantized, also extends within these boundaries. The training vectors for the quantizer design specify the users' angles $\boldsymbol{\psi}_i = [\phi_i\ \ \theta_i]^{\text{T}}$. We now turn to the distortion measure that determine the cost criterion for the quantizer design. In the ideal settings (e.g., infinite codebook entries), the maximum attainable absolute array factor $\sqrt{N_x N_y}$ could be achieved at any user position. However this is not realizable in practice, because the number of possible user angles is vastly larger than the finite codebook size. Hence, define the distortion measure between the ith vector $\boldsymbol{\psi}_i$ and the jth codebook entry $\boldsymbol{\chi}_j$ as,

$$d(\boldsymbol{\psi}_i, \boldsymbol{\chi}_j) = \sqrt{N_x N_y} - |A_{\text{tx}}^{(x)}(\boldsymbol{\psi}_i, \boldsymbol{\chi}_j)||A_{\text{tx}}^{(y)}(\boldsymbol{\psi}_i, \boldsymbol{\chi}_j)|. \tag{12}$$

In other words, the distortion between ith user and jth beam steering angle is *the absolute array factor drop* from the maximum achievable array factor. This will subsequently take into account the varying beam width.

Next, a variant of GLA is derived to find the optimal codebook of beam steering angles. Each algorithm iteration (called Lloyd iteration) consists of the following two main steps:

1. Fix codebook entries, i.e., the beam steering angles, and assign each training vector (user) to the codebook entry achieving the least distortion. This is, in fact, the clustering partition (nearest neighbor) operation. Let $\mathcal{S}_j$ be the set of users assigned to codebook entry j (the jth cluster). The cluster association rule is according to,
$$\mathcal{S}_j = \{i \colon d(\boldsymbol{\psi}_i, \boldsymbol{\chi}_j) \leq d(\boldsymbol{\psi}_i, \boldsymbol{\chi}_k),\ \forall k \neq j\}. \tag{13}$$
2. Fix the training vectors partition into clusters, then optimize the codebook entries to minimize the average distortion seen in each cluster. Codebook entries are optimized as,
$$\boldsymbol{\chi}_j = \arg\min_{\boldsymbol{\zeta}} \frac{1}{|\mathcal{S}_j|} \sum_{i \in \mathcal{S}_j} d(\boldsymbol{\psi}_i, \boldsymbol{\zeta}), \tag{14}$$
where $|\mathcal{S}_j|$ denotes the cardinality of the set $\mathcal{S}_j$. Numerical search or gradient descent algorithms with multiple initialization points can be employed to solve the minimization problem of (14).

At any Lloyd iteration, the average distortion of the codebook is given by,

$$D = \frac{1}{|\mathcal{S}_1 \cup \mathcal{S}_2 \cdots \cup \mathcal{S}_{N_b}|} \sum_{j=1}^{N_b} \sum_{i \in \mathcal{S}_j} d(\boldsymbol{\psi}_i, \boldsymbol{\chi}_j), \tag{15}$$

where $\cup$ is the set union operation. It can be observed that the two main steps of Lloyd iteration guarantees a non-increasing D, which verifies the convergence of this iterative method. Additionally, note that as $N_b \to \infty$, the codebook average distortion $D \to 0$, which supports the validity of the distortion measure considered.

Note that, even-though quantization of $(N_{tx} \times 1)$ beamforming vectors using GLA has been studied in [17, 18], the proposed GLA-based beamforming codebook is designed using only 2-dimensional space. This dimensionality reduction suggests that GLA, in the proposed beam steering settings, is more immune to be trapped in local minima. Furthermore, in practical systems, obtaining 2-dimension user location statistics is highly preferred when compared to the acquisition of high dimension channel statistics due to the complexity of the latter. Any mismatch between training set and actual statistics leads to degradation in system performance.

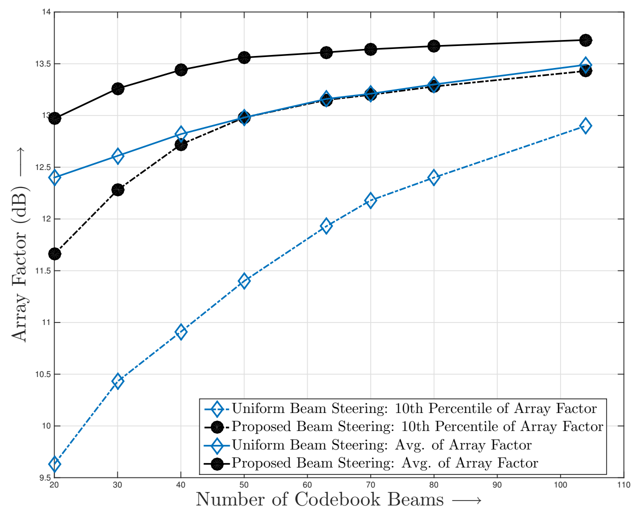

Fig. 3. Beam steering performance comparison. Planar array size 5×5; field-of-view $\phi \in [-180^o, 180^o]$ and $\theta \in [0^o, 90^o]$.

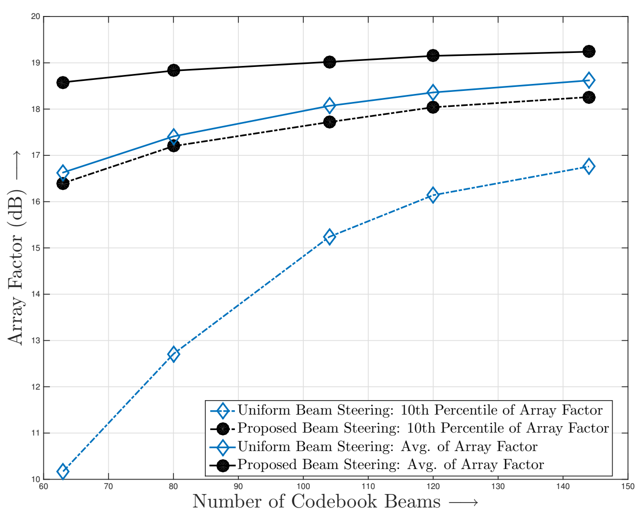

Fig. 4. Beam steering performance comparison. Planar array size 10×10; field-of-view $\phi \in [-180^o, 180^o]$ and $\theta \in [0^o, 90^o]$.

5. NUMERICAL RESULTS

The performance of the designed codebook is assessed by computing average power array factor seen over all user positions. First, an indoor wireless LAN system is considered, where APs are mounted on the ceiling. The ceiling height is assumed to be 3m. The inter-distance between APs is 20m. The stations are uniformly distributed on the horizontal plane. The resolution of possible station positions is 10^4 per cell. Two antenna array configurations at the AP are considered; $N_{\text{tx}} = 5 \times 5$, or $N_{\text{tx}} = 10 \times 10$.

The performance of uniform quantization of angular space into beam steering angles is studied in comparison with the proposed Lloyd-based beam steering approach. The average power array factor as well as its 10th percentile are plotted in dB versus the number of codebook beams N_b in Fig. 3 and Fig. 4. Note that the proposed approach offers gains of up to 2 dB and 6 dB in the average power array factor, and its 10th percentile, respectively. The improvements are pronounced when the codebook size is small, in this case the beam steering method is critical. Furthermore, it is worth noting that, for a given codebook size, the gains due to codebook optimization increase with array size.

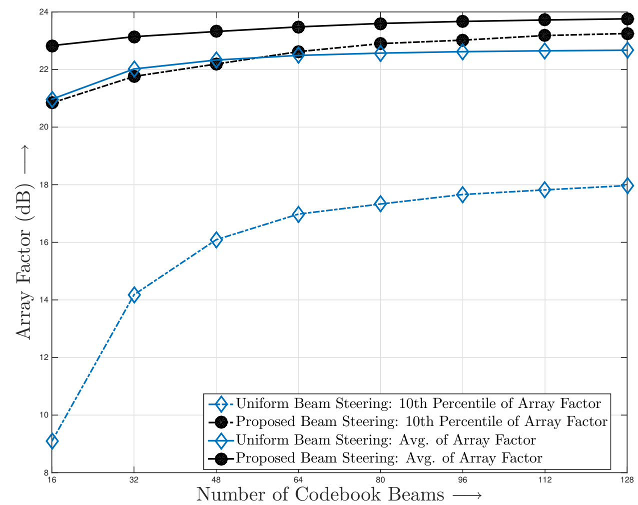

Fig. 5. Beam steering performance comparison. Planar array size 32×8; field-of-view $\phi \in [-60^o, 60^o]$ and $\theta \in [0^o, 90^o]$.

Next, the performance of the proposed Lloyd-based beam steering approach is studied for outdoor cellular 5G systems. The gNB is assumed to be sectorized into 3 sectors (120 degrees per sector). The inter-site distance is 200m, and the gNB height is 10m. Similar to indoor settings, the UEs are assumed to be uniformly distributed over the horizontal plane, and the number of possible UE positions per sector is 10^4. The gNB are equipped with 32×8 planar array. Additionally, no mechanical tilt of planar array is assumed. The average power array factor and its 10th percentile are plotted in dB versus the number of codebook beams in Fig. 5. The proposed beam steering approach offers gains of up to 2 dB and 12 dB, in the average power array factor and its 10th percentile, respectively.

It is worth noting that the performance gain of the proposed beam steering approach is achieved at no cost, this is because the beamforming codebooks are computed off-line and stored in memory. Hence, the system performance is not affected by the complexity of the codebook design. Note further that although the experiments were designed under the assumption of uniform user distribution in space, the algorithm can optimize the codebook to actual user statistics that may be collected over time.

6. CONCLUSION

This paper studies the problem of finding the optimal beam steering angular directions for millimeter wave systems. A novel beam steering approach, leveraging a variant of the generalized Lloyd algorithm, is proposed. Numerical results show compelling improvements when compared to the conventional uniformly distributed steering angles. The proposed approach offers up to 2 dB and 12 dB gains in the mean and the 10th percentile of the power array factor, which ultimately boost the received Signal-to-Noise-Ratio (SNR). The gains obtained by the proposed beam steering approach are pronounced for small codebook sizes or large antenna arrays. It is worth noting that these gains are attained at no additional operational cost, because the codebooks are designed off-line and stored in read-only-memories.

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