Unequally Protected Multistage Vector Quantization for Time-Varying CDMA Channels

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***Abstract*—We present a source-channel coding system for operation over code-division multiple-access channels with time-varying conditions. The proposed system consists of a multistage vector quantizer (MSVQ) in conjunction with unequal protection against channel errors. The receiver estimates the channel conditions and decodes as many stages of the quantizer as can be reliably decoded. The approach to system design and optimization is first derived and evaluated for a system that employs hard decoding of stage indices. The approach is then extended to the more general case of weighted decoding. Simulation results are given for transmission of Gauss–Markov sources over broadcast and slow fading channels. Consistent and substantial improvement is achieved over the standard MSVQ with equal error protection, and the gains, in terms of source signal-to-noise ratio, are in the range of 3–5 dB.**

***Index Terms*—Code-division multiple access, joint source-channel coding, time-varying channels, unequal error protection, vector quantization.**

I. Introduction

THIS WORK is concerned with the design of a joint source-channel coding system for operation over slowly time-varying channels. We make the following assumptions. 1) The transmitter has no access to the exact channel condition, but has knowledge of *a priori* statistical characterization of the channel condition (e.g., probability distribution of the level of attenuation in the channel). 2) The receiver has access to information about the current state of the channel and uses it during the decoding process. Communication scenarios that motivate this problem with the above assumptions include the following: 1) broadcast channel and 2) mobile communication channel without a feedback path from the receiver to the transmitter.

In the case of broadcast applications, a single transmitter transmits an encoded signal to many receivers. Depending on its location and equipment, each receiver experiences a different channel condition in terms of the received signal strength and the level of interference (and noise) power. An appropriate source-channel coding strategy for this case consists of employing a multiresolution source coder followed by unequal protection of its different bit streams [13]. The coarse level bit stream is given the heaviest protection while the finer resolution bit streams are given lighter protection. Each receiver estimates its channel condition and decodes as many bit streams as can be reliably decoded. Thus, the coarse level bit stream can be decoded even when the channel conditions are relatively poor. The receivers that experience better channel conditions will also decode the finer resolution bit streams and will reproduce the signal with higher fidelity.

In a mobile communication scenario, the information about the time-varying channel characteristics is often available at the receiver, and can be used during the decoding process. However, if a feedback path is not available (or is not feasible), the transmitter has no access to this information. Hence, the encoding operation should take into account the range of possible channel conditions that the receiver may experience. Clearly, this situation is quite similar to the broadcast case. Here, too, it is advantageous to adopt a source-channel coding scheme which consists of a multiresolution source coder with unequal protection of the different resolution bit streams.

Theoretical foundations for the work in this area were laid by Cover in his classic 1972 paper on broadcast channels [1]. Subsequent research includes the work of Ramchandran *et al.* [13] and that of Kozintsev and Ramchandran [7], [8]. In [13], the authors describe the design of a subband image coding scheme coupled with unequal error protection (UEP) via multiresolution quadrature-amplitude modulation constellations. In [7] and [8], these ideas are extended to address the mobile communication scenario without a feedback path from the transmitter to the receiver. The approach we adopt in this work is similar in spirit to the above prior work.

We consider a system consisting of a multistage vector quantizer (MSVQ) followed by unequal protection of the different stages. MSVQ is a structured vector quantizer, which is widely used, most notably, in speech coding. MSVQ performs successively refined quantization of the source vector, where the early stages produce a coarse approximation of the source signal, and later stages provide finer detail. The early stages are given heavy protection to ensure decodability even under poor channel conditions. Later stages, which are given lighter protection, are decoded when the channel is cleaner, and enhance the quality of the reproduced signal. The particular scheme we employ for unequal protection of the stage indices is inspired by earlier studies on the design and practical feasibility of a simple UEP scheme called transmission energy allocation, see [3], [2], and [6].

One of the earlier publications addressing the problem of MSVQ design for noisy channels is by Phamdo *et al.* [11] who considered the design of an MSVQ for operation over a *fixed*

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channel with characteristics known to both the transmitter and the receiver. The design of each quantizer stage was optimized given the codebooks of the previous stages. While this design method can impart somewhat higher protection to the initial VQ stages through manipulation of the quantizer partition, it is noteworthy that no attempt is made to "reallocate protection" from one stage to another via explicit optimization. An important difference between this work and the current paper is that here we are concerned with MSVQ-based communication system design for operation over time-varying channels. In particular, we focus on the case where the current channel condition is known to the receiver but not to the transmitter. For this reason, unequal protection of the stage indices is central to our approach.

To concretize the basic approach, we explicitly focus in this paper on unequal protection methods that are appropriate for the code-division multiple-access (CDMA) scenario. However, the ideas are more generally applicable and can be easily extended to other spectrum sharing methods such as time-division multiple access and frequency-division multiple access.

This paper is organized as follows. We begin with a description of the overall system, which consists of MSVQ and unequal protection channel coding. We then describe new MSVQ codebook search and design procedures that exploit the advantages of UEP. Experimental performance evaluation is given for the examples of broadcast and fading channels. It demonstrates the achievable gains in performance over the standard scheme that employs equal error protection (EEP). We then describe and demonstrate how the performance of the scheme is further enhanced by the employment of weighted decoding.

II. Source-Channel Coding with Successively Refined Quantization

In this section, we introduce some of the important concepts that will be used in the paper. Specifically, we will focus on providing a general description of 1) successive refinement quantization and 2) ideas pertaining to UEP.

Successively refined quantization of a d-dimensional source vector x produces a set of indices: $(I_1, I_2, \ldots, I_k)$, where I_i is an r_i bit index (or codeword). Successive refinement implies that given a subset of indices $I_1, \ldots, I_i$, we can generate the reproduction vector $y_i(I_1, \ldots, I_i)$, where the distortion $d(x, y_i)$ is monotone decreasing with i. Specifically, index I_1 provides the basic, coarse quality vector reproduction, while indices $I_2, \ldots, I_k$ provide enhancement via increasing levels of refinement. Thus, $I_1, I_2, \ldots, I_k$ is a list of indices in decreasing order of importance.

The indices are unequally protected against errors, and transmitted on a Gaussian channel with time-varying level of channel gain.[1] By UEP, we mean that each index may be provided protection that is sufficient for a different level of channel gain. Let us denote by α_i the minimum level of channel gain needed to decode a (protected) index I_i. Since index I_1 is the most important, it is naturally provided with the heaviest protection, followed by successively lower levels of protection for the remaining indices. Hence, we have $\alpha_1 \leq \alpha_2 \cdots \leq \alpha_k$.

[1]The term "channel gain" is used in the standard general sense that covers the case of channel attenuation.

At the receiving end, we estimate the current level of channel gain α and decode as many indices as can be reliably decoded. Specifically, if the gain level is such that $\alpha_i \leq \alpha < \alpha_{i+1}$, we decode indices $I_1, I_2, \ldots, I_i$ and obtain the corresponding reproduction vector $y_i(I_1, I_2, \ldots, I_i)$. Intuitively, the objective of UEP is to enable successful decoding of the most important index (or indices) under poor channel conditions, thus ensuring the availability of a crude quality reproduction. As the channel conditions improve, more and more of the remaining indices can be decoded to yield an improved reproduction.

The implementation of the above successively refined vector quantizer requires the storage of $\sum_{i=1}^{k} 2^{r_1+r_2+\cdots+r_i}$, reproduction vectors. Obviously, the storage requirements grow rapidly with the rate and become impractical even for relatively modest applications. Similarly, the complexity of the corresponding encoding operation, which consists of selecting an appropriate set of k indices to be transmitted over the channel, is impractical. The encoding and storage complexity strongly motivates the imposition of a low complexity structure on the vector quantizer. A natural and common choice, which we adopt in this work, is the MSVQ.

III. System Components

This section covers the specific UEP technique and the particular successive refinement quantizer for the communication system under consideration.

A. UEP

Let $B_1, B_2, \ldots, B_k$, respectively, denote the bit streams corresponding to indices $I_1, I_2, \ldots, I_k$. If R_x is the number of source vectors quantized per second, then the bit rate of B_i is related to the number of bits per index r_i, via $R_i = r_i R_x$. Following the notation introduced in the previous section, we denote by α_i the minimum level of channel gain required for B_i. In other words, B_i should be decodable with bit-error rate (BER) below a prespecified acceptable value, as long as the channel gain α is greater than α_i, i.e., $\alpha \geq \alpha_i$. Whenever necessary for obtaining numerical results, we will assume that BER of 10^{-3} is "acceptable."

It is well known that the available radio spectrum can be shared by several users via different multiple-access techniques. Consequently, the UEP method will differ depending on the choice of multiple-access technique. Our focus in this paper is on CDMA. We thus consider the design of a UEP scheme that is appropriate for a CDMA communication system.

1) Capacity Analysis for UEP in CDMA: Consider the transmission of bit streams of rates $\{R_i\}$ over a channel bandwidth of W Hz, with additive noise/interference that is white over the frequency band of interest and of power N_0 per hertz. In CDMA, the entire spectrum is used by each bit stream for communication. We impose the restriction that the different bit streams of the same user will be transmitted using perfectly orthogonal spreading codes. We denote by E_i the transmission energy (power) per hertz employed by the bit stream B_i. Given channel gain α, the received channel signal-to-noise ratio (SNR) is $E_i\alpha/N_0$. We require that the bit stream B_i be decoded whenever $\alpha > \alpha_i$, or equivalently, B_i should be decodable whenever the channel SNR is greater than $E_i\alpha_i/N_0$. If we assume that one can employ perfect channel coding (neglecting the associated complexity and delay requirements), we can relate R_i, W, and channel SNR via the well-known capacity formula

$$R_i = W \log_2\left(1 + \frac{E_i\alpha_i}{N_0}\right). \tag{1}$$

Clearly, given rate R_i, bandwidth W, protection specification in terms of the channel gain value α_i, and noise level N_0, the prescribed protection for different bit streams can be provided by choosing an appropriate value for the transmission energy E_i. This, in turn, can be achieved in the following two ways.

A) Directly transmitting each bit stream with a different value of transmission energy.

B) First encoding the bit streams with binary channel codes of different rates, followed by subsequent transmission of the coded bit streams with equal transmission energy *per channel bit*. By optimizing the rate of the channel code employed for each bit stream, we effectively transmit each bit stream with a different value of E_i.

In this paper, we will adopt the approach A) above. Note that we have assumed that the different bit streams of the same user can be transmitted simultaneously over the entire spectrum while allowing independent reception. In practice, this can be achieved by using orthogonal waveforms to transmit different bit streams of a user as described in the next subsection.

2) Transmission Energy Allocation for CDMA: In Section III-A-1, we motivated the general idea of transmitting each bit stream at a different level of transmission energy. We now develop this idea into a practical UEP scheme for a CDMA communication system.

For reasons that will shortly become evident, we will assume that the rate of each bit stream R_i divides some fixed number R_0. We begin by encoding each bit stream with a suitable binary channel code. Although the code may be different for each bit stream, for the sake of simplicity, we employ here a common channel code, and specifically a rate-$1/2$ convolutional code. For bit stream B_i, the channel code produces a sequence of binary symbols $\{v_i(j)\}$, where $v_i(j) \in \{+1, -1\}$. Since the rate of the code is $1/2$, the symbol rate is $2\,R_i$ symbols/s. We provide unequal protection by transmitting the symbols from different bit streams at differing levels of transmission energy. Let e_i denote the *energy per information bit* employed for transmission of bit stream B_i, that is, each one of the channel symbols $\{v_i(j)\}$ is transmitted with energy $e_i/2$. Let $V_i(t)$ denote the signal

$$V_i(t) = \sum_j \sqrt{\frac{e_i}{2}} v_i(j) sq_i(t - j\,T_i)$$

where $T_i = 1/2\,R_i$, and $sq_i(t)$ is a square pulse given by

$$sq_i(t) = \begin{cases} 1, & \text{if } 0 < t \le T_i \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

To spread the signal $V_i(t)$, we employ a pseudonoise waveform $pn_i(t)$ given by

$$pn_i(t) = \sum_l PN_i(l)\,\delta(t - lT_c)$$

where T_c is the chip duration, $\delta(\cdot)$ is the Dirac delta function, and $\{PN_i(l)\}$ is a binary pseudonoise sequence with $PN_i(l) \in \{+1, -1\}$. The pseudonoise sequences are designed such that they are orthogonal in the following sense. Consider the segments of the pseudonoise sequences corresponding to the jth symbol of each bit stream: $\{PN_1(l_1), PN_1(l_1 + 1), \ldots, PN_1(l_2\}, \ldots, \{PN_k(l_1), PN_k(l_1 + 1), \ldots, PN_k(l_2)\}$, we have

$$\sum_{l=l_1}^{l_2} PN_i(l)\,PN_{i'}(l) = 0, \qquad \text{for all } i \ne i'. \tag{3}$$

We require (3) to hold true for segments of pseudonoise sequences corresponding to every symbol.

Pseudonoise sequences that meet these requirements can be designed via the following technique. We start with a pseudorandom sequence $\{PN(l)\}$ generated using maximum length shift register (MLSR), e.g., [16]. This MSLR sequence is delayed by a time interval corresponding to L chips to obtain the sequence $\{PN(l-L)\}$. It is important that each user employ a different value of L. Since the rate of each bit stream is assumed to divide R_0, the rate of each coded bit stream will divide $2\,R_0$. It follows that the number of chips per symbol duration for bit stream B_i, given by T_i/T_c, is an integral multiple of $C_0 = 1/2\,R_0\,T_c$. We design k orthogonal sequences of length C_0. If C_0 is a power of 2, this can be readily achieved via Hadamard sequences[2] (see, for example, [16] for details). We now group the pseudorandom sequence $\{PN(l-L)\}$ into successive groups of C_0 chips. To obtain $PN_i(l)$, we simply multiply each group of C_0 chips in $\{PN(l-L)\}$ with the ith orthogonal sequence. It can be easily verified that the resulting sequences $\{PN_i(l)\}$ satisfy (3).

It is important to note that while the signals corresponding to bit streams transmitted by the same user will be perfectly orthogonal, the received signals from different users will exhibit correlation. To despread the received signal of a particular user, the receiver will employ the MSLR sequence delayed by a value L corresponding to this user. Also, the receiver is synchronized to cancel the effects of propagation delays experienced by the signals of this user. Hence, the received signals of other users will not be orthogonal to the signals of the user targeted for decoding. Moreover, the correlation between the received signals of other users and that of the targeted user is approximated by a Gaussian random variable. This is the primary source of channel noise.

We transmit over the channel the signal $s(t)$ given by

$$s(t) = \left(\left\{\sum_{i=1}^{k} V_i(t)\,pn_i(t)\right\} * p(t)\right)\cos(\omega_c t)$$

[2]It is assumed that $k < C_0$.

where "$*$" denotes convolution, $p(t)$ is a spectrum shaping pulse, and ω_c is the carrier frequency. The pulse $p(t)$ is assumed to be scaled appropriately to ensure that each symbol in the ith bit stream is transmitted with energy $e_i/2$.

At the decoding end, after demodulation, we receive $r(t) = \sqrt{\alpha}s(t) + n(t)$, where $n(t)$ is Gaussian channel noise with variance N_0, and $\sqrt{\alpha}$ is the channel gain.

The level of protection provided to each bit stream can be determined as follows. Let γ^* denote the minimum value (not in decibel units) of the channel SNR (CSNR), which ensures that the decoded BER is below the "acceptable" value.[3] It is easy to see that the level of CSNR for bit stream B_i is related to its transmission energy level e_i, the variance of the channel noise N_0, and the channel gain α, through $\gamma = \alpha e_i/N_0$. Thus, the minimum level of channel gain that bit stream B_i can withstand is given by $\alpha_i = \gamma^* N_0/e_i$.

B. Multistage VQ-Basic Structure

A k-stage MSVQ consists of k codebooks, $C_1, C_2, \ldots, C_k$. Codebook C_i is a set of 2^{r_i} codevectors that are addressable by an r_i bit index, I_i. A given source vector x is approximated by

$$\hat{x} = u_1(I_1) + u_2(I_2) + \cdots + u_k(I_k) \tag{4}$$

where $u_i(I_i)$ is the codevector in C_i that is designated by index I_i. The objective of the encoding operation is to select a codevector from each codebook such that the error $d(x, \hat{x})$ is minimized, where $d(\cdot, \cdot)$ is a distortion measure. The set of indices $(I_1, I_2, \ldots, I_k)$ is transmitted and allows the decoder to reproduce $\hat{x}$. Ideally, we would like to have the encoder perform an exhaustive search to find the best combination of indices $(I_1, I_2, \ldots, I_k)$ for transmission. However, this computation is often prohibitively complex and, instead, one may adopt a procedure called M–L search that approximates the optimal exhaustive search, (see, e.g., [9]).

M–L Search: We first compare the source vector x with all codevectors in C_1 and select the L vectors that best approximate x in the sense of the given distortion measure $d(\cdot, \cdot)$. These are the L "survivors" at stage 1 and are denoted by $\{u_1^{(l)}, l = 1, \ldots, L\}$. In the next step, we consider all possible combinations of a survivor (from the L available) and a codevector from C_2. Of all possible combinations, we select the "best" L combinations for approximating x. These are the L survivors at stage 2, and are denoted by $\{u_2^{(l)}, l = 1, \ldots, L\}$.[4] The survivor selection procedure is repeated until we reach the final stage M, where instead of selecting the best L combinations, we select the single combination that best approximates the source vector x. This combination, which is the winning path in the corresponding trellis, determines $\hat{x}$.

IV. Design Problem Statement

We consider the communication of source vector x over a slowly time-varying channel, and fix the total energy available for transmission of each source vector at E_{tot}. The justification for fixing the total energy for transmission each source vector stems from the fact that in a CDMA scenario, the signal power of one user contributes to the noise power of the other users. Hence, fixing (or upper bounding) the transmission energy employed by each user ensures fairness to all users.

A k-stage MSVQ is employed for quantization. Each of the stage codebooks are assumed to have 2^r codevectors. In other words, all the stage indices are r-bit long. The size of the codebooks 2^r is determined by limitations on the encoder search complexity, and the memory available for codebook storage. In the present problem, we will assume that the parameters k and r are given and fixed. At first sight, it may seem that fixing k would severely restrict the achievable performance. Here, however, we allow the user the flexibility of distributing the transmission energy unequally among the different MSVQ stages. Thus, if a user opts for maximum robustness, the transmission energy can be entirely allocated to the first VQ stage. On the other hand, if minimal quantization error is required, the transmission energy is equally distributed among the k VQ stages. In between these two extremes lies a tradeoff range (and the focus of this design problem) where some of the VQ stages are transmitted with more energy than others. The precise mechanism to implement such a scheme has already been detailed in Section III-A-2.

Let e_i denote the energy per bit allocated for transmission of the ith stage index. We specialize our prior discussion of unequal allocation of transmission energy to the case of VQ stages as follows. Let α_i denote the minimum channel gain required to assure reliable decoding of the ith stage indices. If γ^* is the minimum CSNR per bit needed to successfully decode an index bit with probability of error below a prescribed value, then (see Section III-A-2) α_i is related to e_i via $\alpha_i = \gamma^* N_0/e_i$, where N_0 is the level of noise (due to multiuser interference) at the receiver end.

The receiver estimates the current level of channel gain α, and decodes as many stage-indices as can be reliably decoded. Thus, if $\alpha_i \le \alpha < \alpha_{i+1}$, the indices $I_1, I_2, \ldots, I_i$ are decoded. The corresponding reproduction is given by

$$\hat{x} = u_1(I_1) + \cdots + u_i(I_i).$$

The design problem is stated as follows. Given a statistical characterization of the level of channel gain α in the form of the probability density function (pdf) $p(\alpha)$, optimize the overall system so as to minimize the expected distortion $\overline{D} = E[||x - \hat{x}||^2]$, while meeting the given constraints on the total transmission energy per source vector: $\sum_i r\, e_i = E_{\text{tot}}$.

V. System Design

The overall system design can be divided into the following three parts:

1) determination of an appropriate codebook search procedure;
2) design of the decoder codebooks;
3) optimization of the UEP scheme, via suitable allocation of the available transmission energy.

[3] It is important to note that the value of γ^* depends *only* on the performance of the channel code. Since our UEP scheme employs one channel code for all bit streams, the value of γ^* is the same for all bit streams.

[4] Note the vector $u_2^{(l)}$ is the sum of two vectors: one from the set $\{u_1^{(l)}, l = 1, \ldots, L\}$ and one from the codebook C_2.

We begin by describing a codebook search technique that is "tailored" to the present problem. We then describe the corresponding design method.

A. Codebook Search

The objective of the encoder is to determine the best set of indices $(I_1, I_2, \ldots, I_k)$ to be transmitted over the channel. The information accessible to the encoder includes the set of decoder stage codebooks $\{C_i\}$, the channel gain thresholds for stage decodability α_i, and the channel gain pdf $p(\alpha)$.

Let us first derive an explicit expression for the expected value of the distortion, given that indices $I_1, I_2, \ldots, I_k$ are to be transmitted. It is convenient to define a set of "stage decoding probabilities" $\{P_i\}$, where P_i denotes the probability that the decoding procedure *stops* at the ith stage, that is, the probability that $\alpha_i \le \alpha < \alpha_{i+1}$. The stage decoding probabilities are given by

$$P_i = \int_{\alpha_i}^{\alpha_{i+1}} p(\alpha) \mathrm{d}\alpha, \qquad \text{for } i = 0, 1, \ldots, k \tag{5}$$

where we set $\alpha_0 = 0$ and $\alpha_{k+1} = \infty$. Note that P_0 denotes the probability that no stage is decoded successfully. We can now write the conditional expected distortion for a given source vector x and a set of stage indices $(I_1, I_2, \ldots, I_k)$ as

$$\begin{aligned} D(x) &= E\left[\|x - \hat{x}\|^2 | I_1, I_2, \ldots, I_k\right] \\ &= \sum_{i=0}^{k} P_i \left\| x - \sum_{l=1}^{i} u_l(I_l) \right\|^2. \end{aligned} \tag{6}$$

Note that the expectation is over the channel statistics, given the source vector, and the transmitted stage indices. Ideally, one would like to evaluate (6) for all possible sets of indices $(I_1, I_2, \ldots, I_k)$, and choose the set that minimizes the expected distortion. However, the complexity requirements of such an exhaustive search are virtually always prohibitive. To circumvent this problem, we employ the following modified M–L search procedure.

Modified M–L Search: The objective here is to find a low complexity approximation to the exhaustive search for the set of indices $(I_1, I_2, \ldots, I_k)$ that minimizes the distortion (6). As described in Section III-B for standard M–L search, the basic idea is to proceed sequentially from stage-1 to stage-k, retaining L survivor combinations at each stage. In principle, one wishes to minimize the overall distortion (6) at each stage. However, at stage J, we do not have information about vector selection of the subsequent stages. We therefore approximate all subsequent stage codevectors by 0 (in general one would use their mean value, but in the case of MSVQ, except for the first stage, the mean is approximately 0). Hence, the encoding cost function at the Jth stage is

$$D_J(x) = \sum_{i=0}^{k} P_i \left\| x - \sum_{l=1}^{i} u_l(I_l) \right\|^2, \quad \text{where } u_l = 0 \text{ for } l > J. \tag{7}$$

Using this cost function, we perform an M–L search. The best codevector combination at the last stage determines the selected codevectors $\{u_i\}$. The corresponding set of stage-indices is transmitted over the channel.

B. Iterative Design

To design the decoder codebooks and the UEP scheme, we start with a training set of source vectors $\mathcal{T} = \{x\}$. We employ an iterative method where each iteration consists of two complementary steps, namely, codebook design (MSVQ design) and transmission energy allocation (UEP design). The goal is to minimize the average distortion

$$\overline{D} = \frac{1}{|\mathcal{T}|} \sum_{x \in \mathcal{T}} \sum_{i=0}^{k} P_i \left\| x - \sum_{l=1}^{i} u_l(x) \right\|^2 \tag{8}$$

where $|\mathcal{T}|$ is the size of the training set $\mathcal{T}$. The following is a high level summary of the overall algorithm.

1) Choose an initial set of MSVQ codebooks and an initial UEP energy allocation scheme.
2) Fix the energy allocation and redesign the codebooks (MSVQ design).
3) Fix the codebooks and reoptimize the energy allocation (UEP design).
4) If the decrease in average distortion $\overline{D}$ is smaller than a prescribed threshold, stop, else go to step 2).

In the next two subsections, we further describe steps 2) and 3)—the two principal steps of the iteration.

1) MSVQ Design Given the Transmission Energy Allocation: Given a training set $\mathcal{T} = \{x\}$, the channel gain thresholds for stage decodability $\{\alpha_i\}$, and the decoder codebooks obtained from the design of the previous iteration (or from the initialization if this is the first iteration), we redesign the decoder codebooks as follows. In each cycle of the iteration, the codevectors of one stage codebook are modified.

1) Set the stage counter $i = 1$.
2) Encode the training set $\mathcal{T}$ and partition it into subsets $\{R_I\}$, $I = 0, 1, \ldots, 2^r - 1$, where R_I consists of training vectors in $\mathcal{T}$ to which the search procedure assigns I as the ith stage index.
3) Adjust the entries of codebook C_i, to minimize $\overline{D}$, while keeping the codebooks $\{C_j\}_{j \ne i}$ fixed. The update formula for the codevectors $\{u_i\}$ of the codebook C_i is

$$u_i(I) = \frac{\displaystyle\sum_{j \ge i} P_j \sum_{x \in R_I} \left(x - \sum_{k \le j, \, k \ne i} u_k \right)}{\displaystyle\sum_{j \ge i} P_j |R_I|} \tag{9}$$

where $|R_I|$ denotes the size of R_I. Equation (9) can be obtained by differentiating (8) with respect to $u_i(I)$, and setting the derivative to zero.
4) If $i < k$, increment i, and go to step 2). If $i = k$, check for a stopping criterion. If criterion not met, go to step 1).

It should be noted that in step 2) we employ the suboptimal M–L search. Although M–L search typically approximates the full search well, its suboptimality implies that full convergence to a local minimum is not guaranteed. This problem is well known for M–L search in general. Our experiments show that this theoretical shortcoming is of minor significance in practice, as even when the algorithm does not fully converge, it experiences a small limit cycle and provides very good designs.

2) UEP Scheme Optimization for a Given MSVQ: Given an MSVQ, we need to design the UEP scheme that provides protection for its various stages. In other words, we need to find the set of transmission energy levels $\{e_i\}$ that satisfies $\sum_i r\, e_i = E_{\rm tot}$, and minimizes the average distortion.

Although one may develop sophisticated methods to optimize UEP, we consider the following simple, low complexity approach that seems to work well.

1) Generate several candidate energy allocations $\{e_i\}$, that meet the constraint on the total transmission energy.
2) Evaluate the average distortion $\overline{D}$ for each set.
3) Choose the energy allocation that minimizes the average distortion.

Evaluation of $\overline{D}$ for a set of energy allocation $\{e_i\}$ levels is a low complexity operation and can be performed as described next. Let D_i denote the average distortion incurred when the decoding is stopped at the ith stage. That is

$$D_i = \frac{1}{|\mathcal{T}|}\sum_{x\in\mathcal{T}}\left\|x-\sum_{j=1}^{i}u_j(x)\right\|^2, \qquad i=0 \text{ to } k.$$

First, we evaluate and store the set of stage distortion values $\{D_i\}$. For a given set of energy allocation values $\{e_i\}$, we determine the UEP channel gain thresholds as $\alpha_i = \gamma^* N_0/e_i$. (Recall that γ^* is the level of CSNR above which the channel code used for transmission of the stage indices provides decoded BER below the prescribed threshold.) We then calculate the set of stage decoding probabilities $\{P_i\}$, using (5). The average distortion $\overline{D}$ can now be evaluated from $\{D_i\}$ and $\{P_i\}$ as

$$\overline{D} = \sum_{i=0}^{k} P_i\, D_i. \tag{10}$$

Note that the term $P_0\, D_0$ accounts for the fact that, with probability P_0, none of the stage indices are decoded.

VI. Results

We designed a four-stage MSVQ for a Gauss–Markov source with correlation coefficient 0.8. A vector dimension of 6 was used. Each stage consisted of 64 codevectors, and hence generated a 6-bit index. As described in Section III-A-2, we consider a transmission scheme where all the index bits are encoded by a rate-1/2 convolutional code. The particular convolutional code chosen for system performance evaluation is the constraint-length 6, maximum free-distance code with generator polynomials listed in [12, Table 8-2-1]. We simulated the performance of this code and observed that the decoded BER is below 10^{-3} as long as the CSNR per (index) bit is above 6 dB, i.e., $\gamma^* = 4.0$. We considered two types of scenarios, broadcast and fading channel, to illustrate the performance of the proposed technique for UEP-MSVQ design. For both channels, the pdf of the channel gain α can be estimated analytically (as will be explained below). Using these density estimates, the UEP-MSVQ scheme is designed as described in Section V.

The training set used for the design consisted of 25 000 vectors. The performance of the system was evaluated over a test set of 10 000 vectors as follows. Each vector in the test set was encoded by the codebook search algorithm of Section V-A. For each test vector, a value of the channel gain α was randomly generated according to the appropriate density. The value of α determines the number of VQ stages that can be successfully decoded for the reproduction, and the resulting distortion can be computed. This distortion, averaged over the test set, was used as the measure for system performance in our experiments.

The performance of UEP-MSVQ was compared to that of an EEP-MSVQ scheme, which distributes the transmission energy uniformly among all the stage indices. It is important to emphasize that in the case of EEP-MSVQ, for a given value of total transmission energy $E_{\rm tot}$, one has the freedom to decide on the optimal number of bits to be employed for quantization. Here, there is an obvious tradeoff between the quantization error and the distortion caused by channel errors. By employing more bits for quantization, the quantization error is reduced at the cost of corresponding increase in the level of channel gain required to successfully decode stage-index bits. Consequently, the distortion caused by the channel errors increases. Thus to ensure fairness of comparison, we considered the performance of several EEP-MSVQ schemes employing overall number of bits per vector in the range of 6–24, with equal transmission energy per bit allocated to each stage index. Here, too, each scheme was designed using a training set of 25 000 vectors and the system performance was estimated with a test set of size 10 000 vectors. At every level of average CSNR, the EEP-MSVQ scheme that yielded the best performance was chosen to represent EEP-MSVQ in the comparison. As one might expect, there was no "uniformly best" EEP-MSVQ scheme. Thus, in our plots, the total number of EEP-MSVQ bits varies with the level of $E_{\rm tot}/N_0$.

A. Broadcast Channel

Let us consider a broadcast scenario where a base station is transmitting a coded signal to several receivers within a circular cell of radius R. The strength (power) of the received signal, at distance r, from the base station is modeled by $E(r) \propto 1/r^k$. The value of k depends on the characteristics of the region. We will use $k = 4$, a typical value for cellular environments [14]. Assuming that the users are uniformly distributed in the cell, the pdf of channel gain at the receiver can be estimated as

$$p(\alpha) = 0.5\sqrt{\frac{\alpha_R}{\alpha^3}}, \qquad \text{for } \alpha \geq \alpha_R$$

where α_R is the channel gain at distance R from the base.

In Fig. 1, the performance of UEP-MSVQ is compared to that of EEP-MSVQ at different values of $E_{\rm tot}\alpha_R/N_0$ (the performance of UEP-MSVQ and EEP-MSVQ is depicted by the UEP-H and EEP-H curves, respectively). The plots indicate that UEP-MSVQ achieves performance gains of about 1 dB over EEP-MSVQ. Note also that the gains are more pronounced at small values of $E_{\rm tot}\alpha_R/N_0$.

The performance gains obtained by UEP-MSVQ over EEP-MSVQ are the result of the *combined* effect of unequal

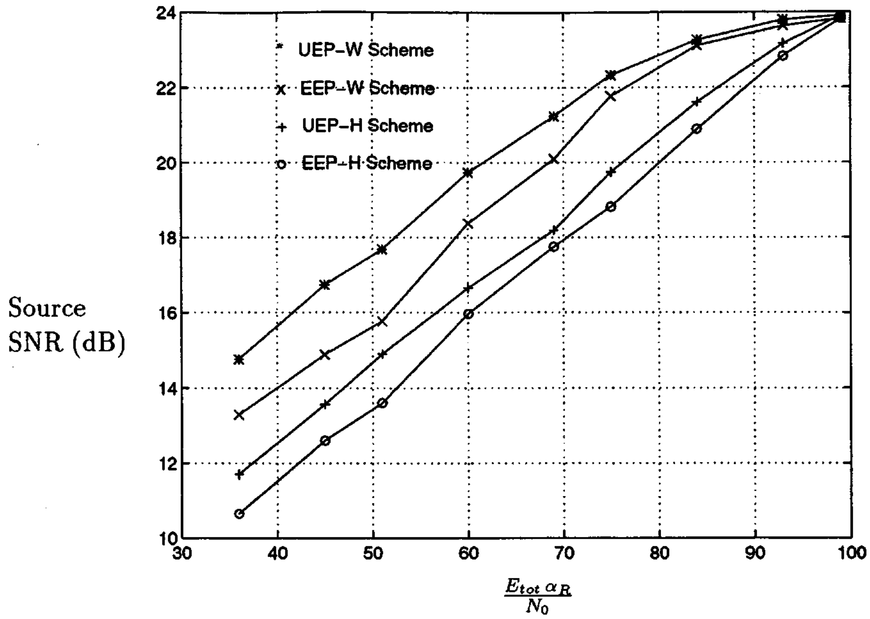

Fig. 1. Comparison of the performance of different MSVQ schemes in a broadcast scenario. UEP-W: UEP scheme with weighted decoding. EEP-W: EEP scheme with weighted decoding. UEP-H: UEP scheme with hard decoding. EEP-H: EEP scheme with hard decoding.

protection and the novel MSVQ search and design method. To illustrate this point and highlight the contribution of the MSVQ search and design method, we consider the performance of a simplistic MSVQ-based UEP scheme. In this scheme, a traditional MSVQ whose design and M–L codebook search do not take into account the stage decoding probabilities $\{P_i\}$ is provided with UEP via the technique described in Section V-B-2. The performance of this scheme was evaluated for selected values of $E_{\rm tot}\alpha_R/N_0$. The corresponding values are tabulated in Table I for comparison with the performance of UEP-MSVQ and EEP-MSVQ. It can be seen that the performance of the simplistic UEP scheme is marginally better than, and occasionally even below, that of EEP-MSVQ. These results clearly emphasize the importance of appropriate MSVQ search and design optimization for exploiting the advantages offered by the UEP coding.

B. Fading Channel

Here, we compare the performance of UEP-MSVQ and EEP-MSVQ over a Nakagami-m fading channel. A Nakagami-m fading model applies to the case of m-path diversity with independent Rayleigh fading on each of the paths. For a CDMA communication system, such diversity is readily obtained when multipath signals of equal strengths are coherently combined (maximum ratio combining). In particular, we consider the case of $m = 2$. For this channel, the received channel gain has the pdf

$$p(\alpha) = \frac{4\alpha}{\overline{\alpha}^2} \exp\left(-\frac{2\alpha}{\overline{\alpha}}\right)$$

where $\overline{\alpha}$ is the average value of the channel gain. In Fig. 2, the performance of UEP-MSVQ and EEP-MSVQ is depicted versus $E_{\rm tot}\overline{\alpha}/N_0$. The curves are labeled UEP-H and EEP-H, respectively. We observe that, in this case, large performance gains of about 2 dB can be achieved by the UEP scheme.

TABLE I

COMPARISON OF THE PERFORMANCE OF THE SIMPLISTIC UEP SCHEME, CONSISTING OF TRADITIONAL MSVQ WITH UEP FOR STAGE INDICES, WITH THE PERFORMANCE OF THE PROPOSED UEP SCHEME. PERFORMANCE OF THE EEP SCHEME IS ALSO INCLUDED IN THE TABLE. RESULTS ARE GIVEN FOR THE CASE BROADCAST CHANNEL DESCRIBED IN SECTION VI-A. ALL THREE SCHEMES ARE BASED ON HARD DECODING OF STAGE INDICES

$\frac{E_{tot}\alpha_R}{N_0}$	36	52	60	75
Overall SNR (dB) EEP scheme	10.6	13.6	16.0	18.8
Overall SNR (dB) simplistic UEP scheme	10.8	13.8	15.6	18.9
Overall SNR (dB) proposed UEP scheme	11.7	14.9	16.7	19.7

We also evaluated the performance of the simplistic UEP scheme described in the last subsection. Here, however, the simplistic UEP scheme captured a substantial portion of the performance gains of UEP-MSVQ over EEP-MSVQ.

VII. WEIGHTED DECODING OF STAGE INDICES

So far we assumed that the stage index was either completely decoded (if the current level of channel gain was above the threshold) or not decoded at all. We, naturally, refer to this as hard decoding. In this section, we explore the possible advantages of employing a weighted decoding rule.[5] The motivation for weighted decoding is derived from the fact that some information may still be extracted from stages that cannot be reliably decoded. In these cases, instead of completely rejecting unreliable indices, we decode them while taking into account their degree of reliability. The advantages of weighted decoding for the case of a full search VQ were described in, e.g., [10], [15], and [3].

[5]It is tempting to use the natural term "soft" decoding, but it should not be confused with the standard interpretation given to this term in channel coding.

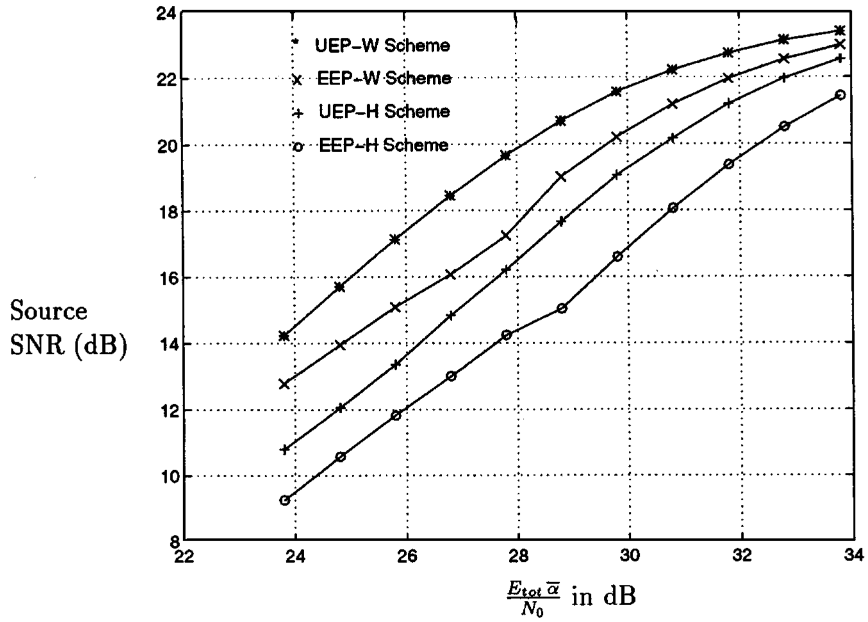

Fig. 2. Comparison of the performance of different MSVQ schemes over a Nakagami-2 fading channel. UEP-W: UEP scheme with weighted decoding. EEP-W: EEP scheme with weighted decoding. UEP-H: UEP scheme with hard decoding. EEP-H: EEP scheme with hard decoding.

Consider the following scheme where we decode *all* the stage indices irrespective of the level of channel noise. For the ith stage, we have been denoting the transmitted index by I_i. Let us denote by J_i the corresponding received index. Depending on the current value of channel gain α, and the UEP channel gain threshold α_i, the index J_i may or may not be a reliable estimate of I_i. Since the decoder knows the transmission energy values employed for the transmission of stage-indices, and also the level of channel noise N_0, it can estimate the error rate in decoding J_i. The reconstruction rule that we propose to use is given by

$$\hat{x} = \hat{u}_1(J_1) + \hat{u}_2(J_2) + \cdots + \hat{u}_k(J_k) \tag{11}$$

where $\hat{u}_i$ is the minimum mean-squared-error estimate of the ith stage codevector given the received stage index J_i and the current value of channel gain α

$$\hat{u}_i(J_i) = E\left[u_i | J_i, \alpha, N_0\right]. \tag{12}$$

We can rewrite (12) explicitly as

$$\hat{u}_i(J_i) = \sum_{I_i} P(I_i | J_i, \alpha, N_0)\, u_i(I_i). \tag{13}$$

Observe that if $\alpha >> \alpha_i$, then the error rate for stage i is low and $\hat{u}_i$ given by (13) approximates $u_i(J_i)$. On the other hand, if $\alpha << \alpha_i$, $\hat{u}_i \approx 0$ (or more generally, the mean of the codebook C_i). Thus, in both cases, the weighted decoding rule simplifies to the hard decoding rule. However, when α is close to α_i, the weighted decoding rule (for the ith stage) significantly differs from the hard decoding rule.

Complexity of Weighted Decoding: The computational complexity of estimating $\hat{u}_i$ is proportional to the size of the codebook C_i, which is 2^r. However, we know that this size is small enough to allow manageable encoding search complexity. Hence the computation of $\hat{u}_i$ using (13) should be feasible. We conclude that weighted decoding is implementable in most practical systems.

A. System Design for Weighted Decoding

Before we demonstrate the performance of weighted decoding, we need to consider the impact of weighted decoding on the UEP optimization and the encoding rule.

The design of the UEP scheme with hard decoding was described in Section V-B-2. The key idea there was to evaluate the average distortion ($\overline{D}$) for several energy allocations and choose the energy allocation that minimizes the average distortion. With weighted decoding, we can employ the same optimization strategy with the following modification in evaluation of the average distortion ($\overline{D}$):

$$\overline{D} = \int E \left\| x - \sum_{i=1}^{k} \hat{u}_i(\alpha) \right\|^2 p(\alpha)\, d\alpha \tag{14}$$

where the expectation in the integrand is evaluated by averaging over the training set.

We can also consider devising an encoding rule which accounts for the fact that weighted decoding is employed at the decoding end. The corresponding mathematical details are summarized in the Appendix to this paper. Such a procedure involves performing an M–L search with a more elaborate cost function. This results in computational requirements that are several times more demanding than the encoding rule of Section V-A. Our experiments indicate that the performance of this modified encoding rule is marginally better than that of the encoding rule which assumes hard decoding. Thus, it appears that the performance does not justify the additional complexity. It is, however, possible that more substantial gains may be achieved in other cases.

B. Performance of Weighted Decoding

In this subsection, we demonstrate the substantial performance gains that can be obtained via the weighted decoding technique. We implemented weighted decoding of stage indices for the design examples of Section VI. (The elaborate encoding rule described in the Appendix was not used.) As in Section VI, the design was performed using a training set of 25 000 vectors. To test the performance of UEP-MSVQ, each vector from a test set (of size 10 000) was encoded and a value of channel gain was generated according to an appropriate pdf. The reconstruction vector was computed using (11). The resulting SNR values are included in Figs. 1 and 2, and the curves are labeled UEP-W. For further reference, we also considered the performance of EEP-MSVQ with weighted decoding. The design and performance evaluation of these schemes was performed as described for EEP-MSVQ in Section VI, except that, during the performance evaluation, the reconstruction vector was computed using (11). The corresponding performance curves are represented by EEP-W in Figs. 1 and 2. We make the following observations.

1) The UEP-W scheme achieves large performance gains in the range of 2–3 dB over the UEP-H scheme for both broadcast and fading channel examples.
2) As a consequence of weighted decoding, the performance of the EEP scheme also improves substantially (by over 2 dB).
3) The overall performance gains of the UEP-W scheme over the standard EEP-H scheme, for low to moderately high values of $E_{\rm tot}$, are in the rough range of 3–5 dB.

These results clearly emphasize the importance of both unequal protection and weighted decoding of the stage indices. It should be reemphasized that both these features can be implemented with only a small increase in the overall complexity.

VIII. CONCLUSION

This work proposes a new approach to the design of a multiresolution (successively refined) vector quantization scheme for operation over time-varying CDMA channels. The motivation for the work stems from its application to signal compression and transmission over broadcast and mobile communication channels. The system consists of an MSVQ whose stages are unequally protected by a simple and easily implementable scheme called transmission energy allocation. The basic idea is to unequally allocate the transmission energy to the VQ stages. We developed a codebook search and design procedure which exploits the varying level of protection provided to the different stages. The performance results were presented for the case of broadcast and Nakagami fading channels. Compared to a standard equal protection scheme, the proposed scheme was shown to achieve substantial performance gains. Finally, we described the weighted decoding technique, where we improve the quality of the reconstructed vector by taking into account the estimates of the decoded BER for each stage index. The implementation of weighted decoding is feasible and results in considerable performance enhancement. The overall gains over standard EEP-MSVQ are in the range of 3–5 dB.

As a final note, it should be reemphasized that the main ideas presented in this paper can be equally applied to non-CDMA communication systems. For such systems, however, the detailed implementation of the UEP scheme requires some obvious modifications. For concreteness, we have specialized the treatment here to CDMA.

APPENDIX
OPTIMAL ENCODING FOR WEIGHTED DECODING

In this appendix, we derive the optimal encoding rule for use in conjunction with weighted decoding. We are given a set of codebooks $\{C_i\}$ and the pdf of the channel gain $p(\alpha)$. The expected overall distortion, if indices $I_1, I_2, \ldots, I_M$ are transmitted, is given by

$$\begin{aligned} &D(x|I_1, I_2, \ldots, I_M) \\ &\quad = \int p(\alpha) E\left[\|x-\hat{x}\|^2 | I_1, I_2, \ldots, I_M, \alpha\right] d\alpha. \end{aligned} \tag{15}$$

The conditional expectation which occurs in the integrand may be written as

$$\begin{aligned} &E\left[\|x-\hat{x}\|^2 | I_1, I_2, \ldots, I_M, \alpha\right] \\ &\quad = E\left[\|x-u_1(J_1)-\cdots-u_M(J_M)\|^2 | I_1, I_2, \ldots, I_M, \alpha\right]. \end{aligned} \tag{16}$$

Let us convert this to a more convenient form, which is similar to so called "noisy channel nearest neighbor rule" [5], [4], as follows:

$$\begin{aligned} &E\left[\|x-\hat{x}\|^2 | I_1, I_2, \ldots, I_M, \alpha\right] \\ &\quad = \left\| x - \sum_i \overline{u_i}\,(I_i, \alpha) \right\|^2 + \sum_i \sigma^2 u_i(I_i, \alpha) \end{aligned} \tag{17}$$

where $\overline{u_i}(I_i, \alpha) = E[u_i(J_i)|I_i, \alpha]$ and $\sigma^2_{u_i}(I_i, \alpha) = E[\|u_i(J_i)\|^2 | I_i, \alpha] - \|\overline{u}_i(I_i, \alpha)\|^2$. The above exploits the known properties of second moments. The overall cost $D(x|I_1, I_2, \ldots, I_M)$ can be obtained by substituting (17) into (15).

We use this new cost in the M–L search procedure to find the optimal stage indices to be transmitted. Note that during the search procedure, the integral has to be approximated by a summation. The complexity of this encoding procedure depends on the number of terms we retain in the summation, and may be considerably higher than that of the search procedure described in Section V-A.

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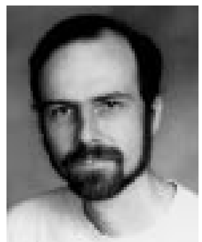

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