NOISY CHANNEL RELAXATION FOR VQ DESIGN

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ABSTRACT

In this paper we study the structure and design of a vector quantizer (VQ) for robust performance under noisy channel conditions. We develop some new insights about the geometrical structure of such a VQ, and introduce noisy channel relaxation - a novel design approach. In particular, the approach is used to attack two basic problems: (1) Optimize the VQ for a given noisy channel; and (2) Optimize index assignment while maintaining optimality for the noiseless channel. For problem (1) we show consistent improvements over descent methods at the cost of manageable increase in complexity (by a factor of up to three). For problem (2) we obtain index assignment with a quality significantly better than known methods, but at computational complexity that grows many times slower.

1. INTRODUCTION

Vector quantization has become a popular and effective technique for signal compression [9]. New "hot" applications, particularly in relation to wireless channels, pose severe constraints both in terms of rate and channel conditions, and gave rise to growing interest in combined source-channel coding. In this paper we concentrate on the problem of designing a VQ with increased robustness to channel errors. Traditionally this problem has been dealt with using two approaches. One approach to tackle this problem involves designing a VQ matched to the statistics of a given channel[1],[2],[6]. The resulting algorithm is an extension of the generalized Lloyd algorithm (GLA) [7] which takes into account the effects of the noisy channel. It is commonly referred to as noisy channel GLA (NC-GLA). However, NC-GLA suffers from the problem of convergence to local minima that are critically dependent upon the initial assignment of binary codewords to codevectors.

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An alternative approach to this problem is that of index assignment. Here the VQ is optimized for a noiseless channel, and robustness to channel errors is achieved by judicious assignment of binary codewords to the codevectors. A known method, the binary switching algorithm (BSA) [3] descends in the index assignment cost and converges to a local minimum. However the computational complexity of BSA increases very rapidly with the size of the codebook, restricting its application to VQ of modest size (a VQ of size 512 will typically require 40/45 hours of run time on Sparc 10).

This paper is organized as follows. In Section 2 we describe the problem of channel matched VQ design. An outline of the NC-GLA is given along with some new insights about the geometry of the resulting quantizer. In Section 3 we introduce noisy channel relaxation (NCR) - a novel technique to avoid poor local minima in channel-matched VQ design. The problem of index assignment is discussed in Section 4, where we show that the NCR approach yields efficient and effective index assignment.

2. CHANNEL-MATCHED VQ DESIGN

Let $X = \{x\}$ be a training set of vectors $x \in \mathcal{R}^d$ representing the source to be quantized. The problem of VQ design involves partitioning the input space $\mathcal{R}^d$ into disjoint encoding regions $R_0, R_1, \ldots, R_{2^n-1}$. If the input source vector is $x \in R_i$, then index i is transmitted through the channel. The channel is assumed to be a binary symmetric channel with bit error probability ϵ. Thus, the probability of receiving index j when index i is transmitted is given by

$$p_{j/i} = \epsilon^{d_H(i,j)}(1-\epsilon)^{n-d_H(i,j)}, \qquad (1)$$

where, $d_H(i,j)$ denotes the standard Hamming distance between the binary representation of i and j. The decoder is a lookup table where the received index is used to select a reproduction codevector y_j from the

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codebook $C = \{y_j\}$. The overall distortion is given by

$$D = \sum_i \sum_{x \in X \cap R_i} \sum_j p_{j/i} d(x, y_j), \quad (2)$$

where, $d(x, y)$ is the distortion measure which will be assumed to be squared Euclidean distance. Let N denote the total number of training vectors in X, and N_i be the number of training vectors in R_i. It can be easily shown that for a given set of encoder partition regions the optimal code book is given by

$$y_j = \frac{\sum_i p_{j/i} P_i \mu_i}{\sum_j p_{j/i} P_i}, \quad (3)$$

where, $\mu_i = \frac{1}{N_i} \sum_{x \in X \cap R_i} x$ and $P_i = \frac{N_i}{N}$. For a fixed codebook C, the encoding operation is specified by the following NC-nearest neighbor rule

$$i = \arg\min_l \{d(x, u_l) + \theta_l^2\} \quad (4)$$

where u_i and θ_i^2 are defined as

$$u_i = \sum_j y_j p_{j/i}$$

$$\theta_i^2 = \left\{ \sum_j p_{j/i} \parallel y_j \parallel^2 \right\} - \parallel u_i \parallel^2 .$$

We can conceptually simplify the encoding rule by increasing the dimensionality of the space by one. Define $d+1$ dimensional "augmented" vectors $\tilde{x} = (x, 0)$ and prototypes $\tilde{u_i} = (u_i, \theta_i)$. The encoding rule (4) now becomes the familiar nearest neighbor encoding rule in the "augmented" space. The prototypes $(\tilde{u_i})$ define Voronoi regions in $\mathcal{R}^{d+1}$. The intersection of these Voronoi regions with the hyperplane $x_{d+1} = 0$, determines the encoding regions $\{R_i\}$. This is illustrated in Figure 1 for the case $d = 1$. Note that not all the Voronoi regions in $\mathcal{R}^{d+1}$ will intersect the hyperplane $x_{d+1} = 0$. If the Voronoi region corresponding to index l does not intersect the hyperplane $x_{d+1} = 0$, then the encoding region R_l is empty, and index l is never transmitted. The system is in effect performing some form of "error correction" by not transmitting certain indices. Also note that the encoding complexity of the channel matched VQ is the same as that of a full-search $d+1$-dimensional VQ. A similar observation on encoding complexity of a channel matched VQ was reported earlier in [6].

NC-GLA involves a repetitive application of the NC-centeroid rule (3) to optimize the code book for a given encoder, and the NC-nearest neighbor (4) rule to optimize the encoder for the given decoder.

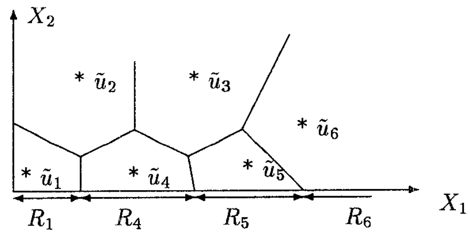

Figure 1: This figure illustrates the formation of the encoding regions defined by the intersection of the Voronoi regions with the line $X_2 = 0$. Note that the regions R_2 and R_3 are empty.

3. NOISY CHANNEL RELAXATION

The effect of the channel on the quantizer design is introduced through the values of $p_{j/i}$ which are determined by ϵ as given by (1). It is known that for small ϵ, the value of $p_{j/i}$ is significant only when $d_H(i, j) \leq 1$. Hence a poor initialization in terms of index assignment often leads to a poor local minimum. However at high values of ϵ the values of y_j and μ_i tend to have a global influence that enables associating codewords that are close in Hamming space with codevectors that are close in the Euclidean space. Figure 2 shows a typical example of the similar solutions produced by different initializations when the channel is very noisy. In fact, the two depicted solutions correspond to an equivalent "error correcting code" as all the codewords of one solution can be obtained from the other by adding the fixed word (0011). Note that such a shift conserves all Hamming distances. We now use this observation to avoid the poor local minima for the final solution. The basic idea is as follows. Instead of applying the NC-GLA with a desired value of ϵ, we start the iterations at a very high value of ϵ and gradually reduce it to the desired value. The solutions obtained at higher values of ϵ, which are less sensitive to initialization, act as effective initial conditions for iterations with lower values of ϵ. Observe that this relaxation is similar in spirit to the idea of annealing but without the computational overload of stochastic moves associated with simulated annealing.

Table 1 summarizes the performance gains that can be obtained using the NCR over NC-GLA. It should be noted that these improvements in performance were achieved with only a modest increase in the computational complexity. In fact the only extra computation comes from the extra iterations, typically 2/3 times, required by NCR over NC-GLA. The relaxation schedule

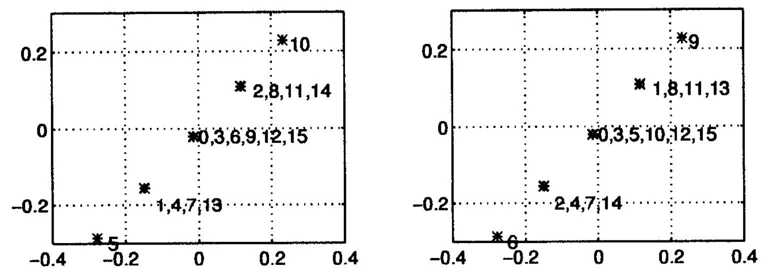

Figure 2: This plot depicts the code vectors along with their binary codewords (represented as integers) obtained for two different initializations using NC-GLA for a binary symmetric channel with transition probability $\epsilon = 0.4$. The 2-dimensional training data was generated by a Gauss-Markov source described by $x(n) = 0.9x(n-1) + 0.44w(n)$ where, $w(n)$ is white Gaussian noise of unit variance. Note that both the solutions have similar location of codevectors while the codewords are shifted by the binary word 0011.

N	16	16	32	32	64	64	128	128
ρ	0	0.9	0	0.9	0	0.9	0	0.9
NC-GLA SNR in dB	4.61	8.64	5.70	9.68	6.35	10.54	7.05	11.45
NCR SNR in dB	4.79	9.14	6.111	9.94	7.06	11.15	7.95	11.85

Table 1: A comparison of results obtained using NCR and NC-GLA. The source was $x(n) = \rho x(n-1) + w(n)$ where, $w(n)$ is white Gaussian noise with unit variance, and vector dimension of 4 was used. The VQ was optimized for a binary symmetric channel with transition probability $\epsilon = 0.01$.

was

$$\epsilon_n = \frac{0.48}{1 + \alpha n^2},$$

with α in the range of 0.01 to 0.005.

4. INDEX ASSIGNMENT

Here we wish to achieve some robustness to channel errors without sacrificing optimality for noiseless channels. Thus the resulting VQ must satisfy the optimality conditions for the noiseless channel, and, the distortion given by (2) simplifies to

$$D = \sum_i \sum_{x \in X \cup R_i} d(x, y_i) + \sum_i \sum_{x \in X \cap R_i} \sum_j p_{j/i} d(y_j, y_i). \tag{5}$$

The problem of index assignment involves judicious assignment of indices to the codevectors to minimize the second term in (5). The corresponding global optimization problem is NP - complete (see [10]). An algorithm to obtain a local minimum based on successive switching of the indices of two codevectors at a time, known as the binary switching algorithm (BSA), was suggested in [3]. However, this algorithm is computationally very demanding for large codebook sizes and hence is limited in its application. For example, doubling the size of the codebook increased the computational complexity of the algorithm by 20/25 times in our experiments.

We suggest the following modification to NCR to handle the problem of index assignment. As described in the earlier section we start the iterations of NC-GLA for a channel with a large value of ϵ. Here, however, we reduce the value of ϵ to zero in the limit. Thus the VQ that we obtain is optimized for the noiseless channel. Moreover the design procedure, that optimizes the cost at progressively lower noise levels, assures that the correspondence between codewords and codevectors at high levels of channel noise is preserved and provides good index assignment at the noiseless limit. As far as computational complexity is concerned, since NC-GLA requires almost the same computational power as GLA, NCR requires about 3/4 times the computation time of GLA. Assuming that GLA had normally been applied to design the VQ for noiseless channel prior to index assignment, we see that NCR combines both in the same design, and provides effective index assignment with an overall design complexity which is clearly manageable.

We demonstrate the promise of NCR for index assignment by comparing it with the performance and computational complexity of BSA. Table 2 presents the results for Gauss iid and Gauss-Markov sources. The schedule followed during relaxation was the same as that mentioned in Section 3. As the value of ϵ is reduced the number of empty encoding regions decreases. When this number reduces to zero and when ϵ is small

N	ρ	BSA	NCR
64	0	8.15/**5.89**	8.16/**6.22**
128	0	9.37/**6.26**	9.38/**6.84**
256	0	10.86/**6.90**	10.76/**7.53**
512	0	12.33/**7.50**	12.21/**8.09**
128	0.8	12.19/**7.90**	12.38/**8.16**
256	0.8	13.35/**7.64**	13.90/**8.40**
512	0.8	14.44/**7.45**	15.40/**8.93**

Table 2: A comparison of the index assignment obtained using NCR and BSA. The source was $x(n) = \rho x(n-1) + w(n)$ where, $w(n)$ is white Gaussian noise with unit variance, and vector d imension of 4 was used. Results are given in the form SNR1/**SNR2**, where SNR1 is the SNR in dB for a noiseless channel and **SNR2** is the SNR on binary symmetric channel with $\epsilon = 0.02$.

enough (say below 0.002) we set $\epsilon = 0$. It can be seen that NCR results in index assignments that are of a superior quality compared to BSA. The computational requirements of the two algorithms are compared in Figure 3. The complexity of NCR grows essentially at the same rate as that of VQ design, while the complexity of BSA grows many times faster. Thus NCR can design VQ (with index assignment) of a size much larger than what can be handled by BSA, given limited time and resources.

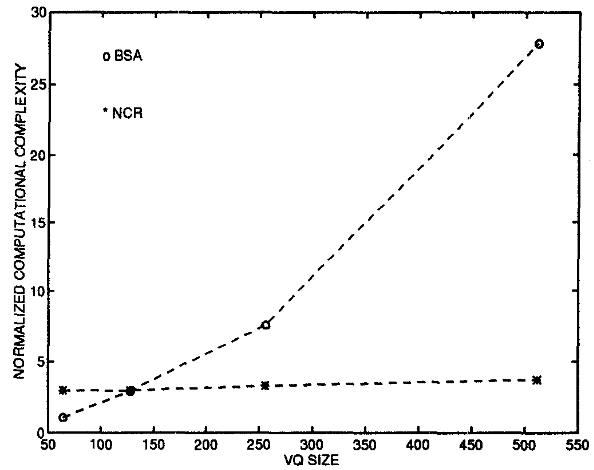

Figure 3: This plot compares the computational complexity of BSA and NCR. The values are given in terms of complexity normalized by the complexity of noiseless channel VQ design.

5. CONCLUSIONS

In this paper we have discussed issues of structure and design of a vector quantizer for operation over a noisy channel. In particular we introduced noisy channel relaxation (NCR) for VQ design. It was shown that for channel-matched VQ design NCR yields a superior performance as compared to NC-GLA with only a modest increase in computational complexity. For the problem of index assignment NCR is shown to outperform BSA both in terms of performance and computational complexity. Although we have demonstrated the use of NCR on simple examples, the technique can be used to build robustness into any signal coding scheme operating over a noisy channel that uses VQ as the quantization block.

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