Jointly Optimized VQ Index Assignment and Transmission Energy Allocation

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Abstract— **In this work we address the problem of designing robust, vector quantizer (VQ) based communication systems for operation over time-varying Gaussian channels. Transmission energy allocation to VQ codeword bits, according to their error sensitivities, is a powerful tool for improving robustness to channel noise. The power of this technique can be further enhanced by appropriately combining it with index assignment methods. We pose the corresponding joint optimization problem and suggest a simple iterative algorithm for finding a locally optimal solution. The susceptibility of the solution to poor local minima is significantly reduced by an enhanced version of the algorithm which invokes the the method of noisy channel relaxation, whereby, the VQ system is optimized while gradually decreasing the assumed level of channel noise. The resulting combined technique is shown to outperform standard pseudo-Gray coding by up to 3.5 dB, and to exhibit graceful degradation at mismatched channel conditions. We conclude with a brief discussion of the impact of TEA on the peak-to-average energy ratio of the transmitted modulated signals.**

I. Introduction

With the advent of wireless personal communication systems, there has been an increasing interest in the area of joint source-channel coding. The challenge lies in the fact that these systems must often work under severe bandwidth constraints, which allow only limited use of explicit channel coding. The time-varying nature of these channels exacerbates matters significantly. A popular method for robust VQ-based communication is index assignment (IA) [9], [10]. Here we employ a source-optimized VQ encoder-decoder pair. This ensures that the system performance is uncompromised in the case of clean channel conditions. Robustness to channel errors is achieved by judicious assignment of indices to the codevectors. The optimization of IA is normally performed for some representative (typical or average) channel condition. However, the resulting IA is known to achieve robust performance over a large range of channel conditions.

In this paper we explore the potential advantages of optimizing the modulation scheme to increase the robustness of VQ based communication systems. While most of the existing literature in the field of combined source-channel coding assumes fixed modulation, notable exceptions include the early work of Bedrosian [1] and the work of Sundberg (see more recent description in [2]), where the idea of transmission energy allocation to the different bits was considered in the context of pulse coded modulation. Recently, it was demonstrated in [3], [6], that the performance of a VQ over a noisy channel can be significantly improved by allocating transmission energy to the VQ output bits according to their sensitivity. An independent work by Ho and Kahn reported similar findings in the context of multicarrier modulation [4], [5].

Motivated by the promising results of our earlier work [3], here we undertake a more detailed investigation into the application of Transmission Energy Allocation (TEA) to a VQ. Another motivation for this work derives from the recent developments in the area of efficient linear power amplifier design, e.g. [7], [8]. These developments allow improved flexibility to experiment with linear modulation methods than was possible in the past. This paper is organized as follows. In section II we briefly review the basic idea of transmission energy allocation (TEA) in the context of a VQ indexed by the natural binary code (NBC) as described in [3], [5]. The motivation behind employing NBC was to obtain varying degrees of error sensitivity for the different bits. This feature is exploited for unequal protection by allocating higher levels of transmission energy to the more sensitive bits. NBC is a convenient heuristic choice for indexing VQ codewords and its combination with TEA provides significant improvements over standard pseudo-Gray index assignment. However, it is not the optimal choice. In section III , we pose the problem of joint optimization of TEA and IA. We develop a direct optimization algorithm for this problem and show that substantial improvements over TEA-NBC can be achieved. The direct optimization method, however, is susceptible to poor local minima that riddle the cost surface due to the complex discrete nature of the IA problem. We tackle this issue by developing an enhanced version of the algorithm that incorporates noisy channel relaxation [12] which has the ability to avoid many shallow local minima. We also include a brief discussion on the impact of TEA on peak-to-average energy ratio of the transmitted modulated signals.

II. Transmission Energy Allocation

In this section we briefly review the basic TEA-NBC method where transmission energy allocation is applied in

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conjunction with the natural binary code as described in [3], [5]. We also include extensions of [3] to handle arbitrary VQ (that are not necessarily source-optimized) and comment on the choice of representative channel conditions.

Consider a source that produces a sequence of independent random vectors x, and the corresponding source-optimized VQ with its codebook $C = \{y_0, y_1, ..., y_{2^n-1}\}$. Given x, the VQ encoder finds the nearest codevector y_I and employs binary modulation to transmit the n-bit index I over a Gaussian channel. The decoder receives a noisy version of the transmitted signal and applies hard decision decoding to obtain the received index J. Note that we could equivalently state that individual bits are transmitted on independent binary symmetric channels, whose bit error rates depend on the corresponding Gaussian channel SNR. Given received index J, the decoder produces the codevector y_J as estimate of the source vector x. Since the VQ is source-optimized, its codevectors satisfy the centroid rule [20]. This implies that the overall distortion can be decomposed into quantization and channel distortion terms:

$$D = E||x - y_J||^2 = E||x - y_I||^2 + \overbrace{E||y_J - y_I||^2}^{D_c}. \quad (1)$$

Index assignment aims at minimizing D_c by a judicious assignment of binary indices to the codevectors (see e.g., [9], [10], [11]). As D_c depends on the channel conditions, IA typically assumes a particular representative (or expected) level of channel noise for the design phase. It is well known that some standard VQ design methods naturally produce relatively good indexing. An important example is NBC which is obtained from VQ design initialized by the splitting method (see e.g. [11]).

Of particular relevance to us is the fact that with NBC the index bits are not equally sensitive to channel errors. To formally define the error sensitivity of bits let us employ a bitwise explicit notation for the transmitted index: $I = (i_1 i_2 ... i_n)$. The sensitivity of the jth bit is defined as the expected amount of distortion caused by a bit error at this location:

$$D_j = E||y_{i_1,i_2,.,i_j,.,i_n} - y_{i_1,i_2,.,i_j^*,.,i_n}||^2, \quad (2)$$

where "superscript *" denotes the complement: $i^* = 1 - i$. In [3] it was demonstrated that the bit sensitivities of a VQ indexed by NBC vary over a large range. We exploit this fact by providing optimal unequal error protection via the allocation of transmission energy to the various VQ bits according to their sensitivities.

As mentioned earlier, the n bits are transmitted independently on a Gaussian channel using binary modulation. Let σ_r^2 be the representative level of Gaussian noise in the channel (i.e., the level of channel noise assumed during the design phase), and let e_j be the energy allocated to the jth bit. Then, the bit error rate is

$$\epsilon_j = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{e_j}{\sigma_r^2}}}^{\infty} e^{-\frac{t^2}{2}}\, dt.$$

Neglecting the probability of more than single bit errors in the index transmission, the distortion due to channel errors simplifies to

$$D_c = \sum_{j=1}^{n} D_j \epsilon_j. \quad (3)$$

We wish to minimize D_c over all choices of $\{e_j\}$, that is, by allocating transmission energy to the n bits, subject to the constraint on the total energy available for their transmission:

$$\sum_{j=1}^{n} e_j = e_{tot}.$$

This constrained optimization problem can be solved using various techniques. The basic idea is to evaluate the set of derivatives $\{\frac{\partial D_c}{\partial e_j}\}$ and use them in either a gradient descent algorithm or a greedy energy quanta allocation algorithm, similar to the one described in [3].

Note: Evaluation of $\frac{\partial D_c}{\partial e_j}$ using (3) is based on single bit error assumption. If the representative level of channel noise is sufficiently high, the probability of multiple bit errors per index will no longer be negligible. For these cases, we proceed as follows to evaluate $\frac{\partial D_c}{\partial e_j}$:

$$\frac{\partial D_c}{\partial e_j} = \sum_I P(I) \frac{\partial P(J/I, \sigma_r^2)}{\partial e_j} ||y_J - y_I||^2, \quad (4)$$

where $P(I)$ is the apriori probability that index I is transmitted (estimated from the training set), and $P(J/I, \sigma_r^2)$ is the probability of decoding index J given transmission of I and channel noise level of σ_r^2.

Performance of TEA-NBC: Table 1 provides a comparison of the performance of TEA-NBC with that of the standard pseudo-Gray (PG) coding. (The table also includes other results that should be ignored for the moment.) The PG method consists of index assignment only, with allocation of equal transmission energy to all the bits. The results are given for a first order Gauss-Markov source. TEA-NBC and PG are optimized for a representative channel SNR of 8 dB, and the performance is tested over the channel SNR range of 4-10 dB. All the values given in Table 1 and elsewhere in this paper depict the performance evaluated over test sets. The results show that in many cases TEA-NBC can achieve large performance gains. The gains are especially impressive under conditions of heavy channel noise, where, for some cases the performance gains can be of the order of 2-3 dB. Thus, exploiting variation in bit sensitivities via TEA is a promising direction.

Remark: The optimization of the transmission energy allocation is performed assuming some representative level of channel noise - σ_r^2. The choice of σ_r^2 can have a significant impact on the resulting robustness of the VQ. If the value of chosen σ_r^2 is too small, the amount of protection provided to the bits is almost the same. Hence, we fail to take advantage of the varying bit sensitivities. An excessively high value for σ_r^2 results in allocation of very little, or no, transmission energy to the least sensitive bits, thereby causing a substantial performance loss when the channel is cleaner. However, a sensible choice of σ_r^2 should reflect the channel conditions that we expect to encounter on the average. If the expected channel conditions translate into too high a σ_r^2, it suggests that the amount of transmission energy available is too low for meaningful transmission of all the VQ index bits. Under such circumstances it might be worthwhile to either increase the total transmission energy (if possible), or reduce the number of bits used by the VQ.

III. Joint Optimization of Index Assignment and Transmission Energy Allocation

While substantiating the promise of TEA, the results of Table 1 also indicate that the choice of NBC though a natural starting point for appling TEA, can not guarantee success. It can be seen that the gains of TEA-NBC diminish with increase in VQ dimension, and with decrease in the correlation coefficient. For example, with $\rho = 0.0$ and dim = 4, PG happens to outperform the TEA-NBC scheme. These observations motivate the search for better methods for combining index assignment with TEA. Another drawback of TEA-NBC lies in the fact that NBC dictates the use of the splitting VQ design method which is itself suboptimal. It is often possible to design better source-optimized VQ by adopting more elaborate techniques such as deterministic annealing [14] or simulated annealing [15]. It is clearly desirable to have a method that exploits the advantages of TEA and yet is generally applicable to any given VQ. This gives further motivation for joint optimization of index assignment and energy allocation.

We first propose a simple and direct approach for joint optimization of IA and TEA. It is based on iterative application of the IA and TEA procedures. However, this technique is susceptible to poor local minimum traps. To attack this problem, we then propose a second, more involved technique which incorporates the idea of noisy channel relaxation [12].

A. Locally Optimal Design

An optimal solution to the problem of joint design of IA and TEA should satisfy the following two straightforward conditions.

Condition 1: Transmission energy allocated to the different bits must be matched to the bit-sensitivities resulting from the underlying IA.

Condition 2: Indexing of the codevectors must be optimal for the underlying transmission energy allocation.

Given any IA, condition 1 can be satisfied by evaluating the bit sensitivities (due to this indexing) followed by optimizing the transmission energies. Condition 2 is satisfied by optimizing the IA for the given energy allocation. In this context we note that many IA algorithms, such as binary switching [9], can be easily modified to take into account unequal bit error rates for the various bits. Hence, such a modified IA technique can be used to optimize the index assignment for a given transmission energy allocation. These observations motivate the following simple strategy for joint optimization of TEA and IA:

1. Start with an initial index assignment.
2. Evaluate the sensitivities of the various bits and apply the energy allocation algorithm.
3. Reassign indices to the codevectors via an IA technique (e.g., binary switching) which has been modified to include the effect of unequal bit error rates.
4. Check convergence (e.g., $\Delta D_c / D_c \leq Th$), if so, stop.
5. Go to step 2.

The algorithm alternates between the TEA and IA steps. Since D_c is monotonically decreasing with each step, we are ensured of obtaining a locally optimal solution to the problem. This algorithm will be referred to as TEA-IA.

We applied TEA-IA to VQ designed for the first order Gauss-Markov source of section II. It was found that the solution depends heavily on the initial index assignment. We experimented with the following two initializations and chose the one that gave the best performance in each case: (a) NBC, (b) NBC followed by pseudo-Gray coding. These results were added to Table 1. The binary switching algorithm [9] was modified to account for the variation in error rates among the bits, and used for optimization of IA. The design assumed channel SNR of 8 dB, while the performance was evaluated at the channel SNR range of 4-10 dB. The iterative optimization of TEA-IA achieves a rearrangement of the code vectors that yields additional modest improvements in performance (up to 0.8 dB). The performance improvements are more pronounced under conditions of heavy channel noise.

It is important to note that larger improvements were achieved wherever TEA-NBC provided little or no gains over the pseudo-Gray method. This observation indicates the importance of joint optimization of IA and TEA.

B. Noisy Channel Relaxation

The significant impact of initialization on the quality of the solution to the joint optimization problem is not surprising, and stems from the well documented local minima problem of IA methods. In the present situation, the susceptibility of the solution to poor local minima is exacerbated with the increased complexity of the joint IA and TEA optimization problem. In fact, our experiments show that some initializations resulted in extremely poor solutions. To overcome this shortcoming we developed a method that has an enhanced capability to avoid these poor local minima.

The problem of poor initialization for IA was addressed by the method of noisy channel relaxation (NCR) [12] where it was suggested to first optimize the VQ for a high level of channel noise, and then gradually reduce the level of channel noise assumed for the design (see also [13] for a related VQ design method). Channel optimized VQ design for a given level of channel noise is performed using the noisy channel generalized Lloyd algorithm (NC-GLA), see e.g., [11]. This gradual reduction of design noise level, or noisy channel relaxation, provides means for avoiding many poor local minima of the IA problem. Initializing the iterations at a very high level of channel noise makes it easier for the system to find a good initial IA which is then tracked and reoptimized as the noise level is reduced. The final iterations are performed for a noiseless channel, thereby yielding a source-optimized VQ albeit with a built-in indexing inherited from the design in earlier stages. The NCR simulation results of [12] demonstrate substantial improvements in the robustness of the VQ to channel errors.

In this work we extend NCR for application to the problem of joint IA-TEA optimization. We start with the observation that NC-GLA can also be modified in a straightforward manner to account for variation in error rates among bits. This modified NC-GLA can therefore be used to incorporate NCR within the joint optimization procedure. To design a VQ via NCR for a given energy allocation, we proceed as follows. We use a very high variance of Gaussian channel noise in the initial iterations and reduce the noise variance as the iterations proceed. At each iteration we use the current level of channel noise to evaluate the bit error rates for the different VQ output bits. Using these values in the NC-GLA method, we perform the channel matched VQ design. The iterations towards the end are performed with zero noise, yielding a "noiseless channel" optimized (i.e., source-optimized) VQ with indexing that takes into account the transmission energy allocated to the various bits.

We now summarize the TEA-NCR algorithm, which integrates index assignment using noisy channel relaxation, and energy allocation, as follows :

1. Initialize the transmission energy allocated to the various bits.
2. Design the VQ using NCR. (The final index assignment can be fine-tuned by the binary switching algorithm.)
3. Evaluate the sensitivities of the various bits and reoptimize the transmission energies allocated to the VQ bits.
4. Check for stopping criterion, e.g., if $\Delta D/D < Th$ stop.
5. Go to step 2.

The overall performance obtained by the TEA-NCR approach is tabulated in Table 1. We can see that TEA-NCR is able to avoid many of the poor local minima that trap TEA-IA yielding improvements of up to 0.8 dB. The overall improvement in performance over standard pseudo-Gray coding is in the rough range of $0.6 - 3.5$ dB under heavy channel noise, while the net improvement over TEA-NBC is in the order of $0.6 - 1.3$ dB.

Details of the relaxation schedule: The iterative VQ design was performed using NC-GLA starting with noise level corresponding to a channel SNR of -5 dB. Under these conditions the number of non-empty encoding regions [18], [12] resulting from channel optimized VQ design is much smaller than the target size of the VQ. The variance of noise was reduced by a factor of 1.01 in each NC-GLA iteration. As the variance of the channel noise is reduced, the number of encoding regions increases. The rate of increase of the number of non-empty encoding regions was found to be an effective means of regulating the relaxation schedule. If the number of non-empty encoding regions increased by more than 2 in a single iteration, the reduction of noise level was stopped. The relaxation was subsequently resumed when the number of non-empty encoding regions remained same over two consecutive iterations. When the number of non-empty encoding regions becomes equal to the target size of the VQ, the noise level was reduced by a factor of 1.1 per iteration.

IV. Conclusions and Discussion

Motivated by the early work of Bedrosian and promising developments in the field of linear power amplifier design, we investigated the idea of transmission energy allocation to provide unequal error protection to the various bits of a vector quantizer according to their importance. We formulated the problem of joint optimization of index assignment and TEA and demonstrated the substantial performance advantages of the resulting optimization. The corresponding solution is however susceptible to poor local minima. To overcome this problem, we suggested a design method based on noisy channel relaxation [12] that has the ability to avoid many shallow local minima. The resulting performance gains over the pseudo-Gray coding range from 0.7-3.6 dB, for the case of Gauss-Markov sources.

A major drawback of TEA stems from the fact that the signal energy employed for transmission of different VQ bits can vary considerably. In other words, the peak-to-average ratio (PAR) of the energy of signals transmitted over different modulation intervals can be high. The PAR resulting from TEA-NCR for the Gauss-Markov example of Table I ranges from 0.3 dB to 2.4 dB. A large PAR increases the linearity requirements of the power amplifier used to transmit the modulated signal. Hence, it is desirable to retain the advantages of TEA while maintaining a small PAR.

It is crucial to note that the above mentioned sizable increase in PAR due to application of TEA was evaluated based on treating the modulation procedure as one dimensional. In practice, we are mostly interested in quadrature amplitude modulation (QAM), where we transmit information on the in-phase and quadrature phase components of a carrier. In the case of binary modulation, the resulting 2-dimensional signals are in the form of 4-QAM constellation. For this case we can drastically reduce the PAR requirements on the 2-dimensional QAM signals by appropriately grouping the bits prior to transmission as follows. We first arrange the n bits of the VQ in decreasing order of sensitivities. If we group the bits i and j for transmission as a single 2-dimensional signal, the corresponding constellation point is $(b_i\sqrt{e_i}, b_j\sqrt{e_j})$, where $b_i, b_j = \pm 1$. The energy contained in this signal point is $e_i + e_j$. Consider the following method of grouping the bits : $(1, n)$, $(2, n-1)$, .., $(\frac{n}{2}, \frac{n}{2}+1)$, where we assume that n is even[1]. The PAR

[1] If n is odd, we consider two consecutive codevectors and group the

Channel SNR		dim = 2			dim = 4		
		$\rho = 0.0$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.0$	$\rho = 0.8$	$\rho = 0.9$
4 dB	PG	4.43	4.63	5.03	3.60	4.52	4.75
	TEA-NBC	5.09	6.40	7.14	2.72	5.01	5.84
	TEA-IA	5.92	6.49	7.16	3.68	5.29	6.29
	TEA-NCR	6.39	7.51	7.79	4.31	6.13	6.64
6 dB	PG	8.12	8.40	8.86	6.11	7.60	8.08
	TEA-NBC	9.36	10.94	11.93	5.51	8.30	9.57
	TEA-IA	10.03	11.05	11.96	6.17	8.39	9.82
	TEA-NCR	10.48	11.88	12.38	6.72	9.09	10.10
8 dB	PG	13.35	13.86	14.43	8.55	11.03	12.15
	TEA-NBC	14.78	16.58	17.78	8.30	11.50	13.22
	TEA-IA	15.27	16.67	17.78	8.60	11.52	13.35
	TEA-NCR	15.56	17.23	18.01	8.87	11.84	13.47
10 dB	PG	18.79	20.18	21.21	9.71	13.03	14.95
	TEA-NBC	19.11	20.97	22.46	9.68	13.07	15.03
	TEA-IA	19.31	20.97	22.46	9.74	13.10	15.16
	TEA-NCR	19.38	21.29	22.35	9.83	13.12	15.16

TABLE I

HERE WE COMPARE THE OVERALL PERFORMANCE OF A VQ PROTECTED AGAINST CHANNEL ERRORS BY THE FOLLOWING TECHNIQUES. PG : PSEUDO-GRAY CODING WITH EQUAL ENERGY ALLOCATED TO ALL THE BITS. TEA-NBC : NATURAL BINARY CODE FOLLOWED BY ENERGY ALLOCATION. TEA-IA : ITERATIVE OPTIMIZATION OF INDEX ASSIGNMENT AND ENERGY ALLOCATION. TEA-NCR : ITERATIVE APPLICATION OF NOISY CHANNEL RELAXATION AND ENERGY ALLOCATION. A GAUSS-MARKOV SOURCE WITH CORRELATION COEFFICIENT ρ WAS USED; VQ SIZE EMPLOYED WAS 256. ALL THE OPTIMIZATIONS WERE PERFORMED AT CHANNEL SNR OF 8 DB AND THE PERFORMANCE IS EVALUATED FOR A RANGE OF CHANNEL SNR OF 4-10 DB.

variation in the energy of the two-dimensional QAM signals, for the case of the TEA-NCR VQ example mentioned earlier, was thereby reduced drastically from 0.3-2.4 dB to 0.02-0.3 dB. Moreover this reduction in PAR is achieved without compromising the robustness of the VQ.

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bits as: $(1,n)$, $(1',n')$, ..., $(\lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil')$. Where, i and i' denote the i-th bit of the two codevectors.