

Transmission Energy Allocation for CDMA Applications

Shrinivas Gadkari and Kenneth Rose*
Dept. of ECE, University of California
Santa Barbara, CA 93106

Abstract

Transmission energy allocation (TEA) refers to redistribution of the available transmission energy among the different bits according to their sensitivity. In this work, we describe the application of TEA to robust vector quantization in a CDMA environment. We consider an interference-limited environment with negligible interference from other cells. In spite of the apparent incompatibility of TEA with such an environment, we show that it is, in fact, highly beneficial. We first analyze the effect of TEA on multi-user interference. We show that optimal TEA design is not separable from the problems of ordering of bits of the various users, and the design of power control. We then prove that the jointly optimal solution to the overall design problem is simple and easy to implement. Finally, we provide experimental evidence to substantiate the performance gains of TEA over the standard index assignment, under conditions of severe fading.

1 Introduction

Consider a source that undergoes encoding and is then transmitted over a channel. It is a simple observation that not all transmitted bits are equally sensitive to channel errors. This fact implies that not all bits require the same level of protection. Transmission energy allocation (TEA) provides unequal protection via an appropriate redistribution of the available quota of transmission energy. Higher level of transmission energy is allocated to more sensitive bits, while less sensitive bits are transmitted with lower energy. The idea of TEA and its application to pulse coded modulation first appeared in the early work of Bedrosian [1] and the subsequent work of Sundberg [2]. Recently the TEA was revisited in the context of robust vector quantizer (VQ) design in [3, 4]. Where, it was shown that TEA can substantially enhance the performance of a VQ operating over a noisy channel.

In this work we show how TEA can be applied to improve the robustness of vector quantization in a multi-user environment. In particular, we consider a CDMA scenario with several users distributed within a cell. We

focus our attention on the user to base-station channel. The key difference between the multi-user environment and the noise limited environment assumed in the earlier TEA work, stems from the fact that, unlike thermal noise, multi-user interference critically depends on the transmission energy employed by the various users. Clearly, the impact of redistribution of transmission energy on the level of interference must be carefully analyzed. To illustrate this point, consider the extreme example where all users transmit bits of the same sensitivity simultaneously. Depending on the sensitivity of the bit being transmitted, all the users in concert, reduce or increase the amount of transmission energy. The total interference power varies in proportion to the energy transmitted per bit, and, the bit error rate, which depends on the ratio of transmission energy to the interference power, remains constant for all bits. In other words, the whole purpose of TEA is defeated. It is therefore obvious that in a multi-user scenario, in addition to the redistribution of transmission energy among different bits, we must consider the ordering and synchronization of bits of the various users. Moreover, we must also account for the so-called near-far problem which results from the fact that received power varies drastically depending on the distance of the user from the base station. This problem is traditionally handled by power control. Obviously, power control and TEA are concerned with the same resource, and cannot be performed independently of each other. We conclude that TEA, power control and ordering of user bits must be jointly optimized.

2 Robust Vector Quantization and TEA

In this section we review TEA in the context of time-varying Gaussian channels. A popular approach to impart robustness to vector quantized data for transmission over time-varying channels is index assignment. The use of source-optimized VQ assures that as long as the channel is clean there is no loss in performance. Robustness to channel errors is achieved by judiciously assigning binary indices to the VQ codevectors.

After index assignment, we still expect the VQ bits to be unequally sensitive to channel errors. We exploit this fact and provide them with unequal error protection. This is achieved by redistributing the available transmission energy to the bits according to their sensitivity. Let β_i denote the sensitivity of the i -th bit, which is defined as the expected distortion caused by a single bit error at the i -th location. Let E_i denote the energy allocated to the i -th bit for transmission using

*This work was supported in part by the National Science Foundation under grant no. NCR-9314335, the University of California MICRO program, ACT Networks, Inc., Advanced Computer Communications, Cisco Systems, Inc., DSP Group, Inc., DSP Software Engineering, Inc., Fujitsu Laboratories of America, Inc., General Electric Company, Hughes Electronics Corp., Intel Corp., Nokia Mobile Phones, Qualcomm, Inc., Rockwell International Corp., and Texas Instruments, Inc..

binary modulation. If σ_n^2 is the variance of the Gaussian noise in the channel (which is allowed to vary with time), then the overall distortion due to channel errors, under single bit error assumption, is

$$D = \sum_{i=1}^k \beta_i \operatorname{erfc} \left(\sqrt{\frac{E_i}{\sigma_n^2}} \right), \quad (1)$$

where, $\operatorname{erfc}(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$.

Since the noise level, σ_n^2 , varies with time, to optimize the energy allocation, we proceed as follows. We select a representative level of channel noise σ_r^2 and distribute transmission energy $\{E_i\}$ to the different bits so that (1) is minimized, subject to the constraint on the total energy $\sum_i E_i = E_{tot}$, where E_{tot} is the energy available for transmission of the codevector. As described in [5], by appropriately choosing a value for σ_r^2 , the robustness of the VQ can be substantially improved over a large range channel noise levels, σ_n^2 .

3 System Description

Consider a source-optimized VQ with k -bits per source vector. Let β_i denote the sensitivity of the i -th bit. These k -bits are transmitted over a multi-path fading channel with CDMA as the spectrum sharing technique (in contrast to the time-varying Gaussian channel described in section II).

Let M denote the number of users in the cell. The I -th user transmits the bit of sensitivity β_{Ii} in the i -th bit interval, employing transmission energy E_{Ii} . Each user uses a different pseudo-noise (PN) sequence to spread the source bits prior to transmission. Let us denote by C the number of chips transmitted per bit. We will assume that the period of the PN sequence is much larger than C (as is common in CDMA systems). The users are at varying distances from the base and each may have a different surrounding environment. The signal of user I , is subject channel gain which we denote by g_I . In the case of mobile communication, the channel gain varies with time. Let us decompose the overall channel gain into two factors as, $g_I = \alpha_I \gamma_I$. We use α_I to denote the slowly varying component of the overall gain, while γ_I denotes the rapidly varying factor. By slowly varying, we mean that α_I remains nearly constant between two consecutive updates of the transmission energy values. The more rapidly varying component, γ_I , can vary drastically between two updates of the solution. It should be mentioned that the rapidly varying channel gain is primarily a consequence of multi-path fading, while the slowly varying gain is due to the effect of propagation path loss, e.g. [7]. In this discussion, we will treat α_I as known, fixed quantities. On the other hand, γ_I are modeled as random variables, which satisfy $\gamma_I > 0$ and $E[\gamma_I^2] = 1$.

Consider the received signal at the base station for the I -th user and in the i -th bit duration,

$$S_{Ii} = \gamma_I \alpha_I \sqrt{E_{Ii}} b_{Ii}, \quad (2)$$

where, $b_{Ii} = \pm 1$ is the value of the transmitted bit. The signals transmitted by the other users act as random

interference given by

$$N_{Ii} = \frac{1}{\sqrt{C}} \sum_{J \neq I} \gamma_J \alpha_J \sqrt{E_{Ji}} W_{IJ}. \quad (3)$$

where, $\{W_{IJ}\}$ are unit variance Gauss iid variables. If the number of users is sufficiently large, we can approximate N_{Ii} as a Gaussian process with variance $\frac{1}{C} \sum_{J \neq I} \alpha_J^2 E_{Ji}$. Assuming that the thermal noise and interference from other cells is negligible, we can write the signal to interference ratio for the i -th bit of this user as

$$SIR_{Ii} = \frac{\gamma_I^2 \alpha_I^2 E_{Ii} C}{\sum_{J \neq I} \alpha_J^2 E_{Ji}}. \quad (4)$$

The corresponding bit error rate (assuming coherent demodulation) is given by $\operatorname{erfc}(\sqrt{SIR_{Ii}})$.

The time-varying nature of the channel gain component γ_I , results in a time-varying value for the bit error rate. A good system design attempts to minimize the bit error rate. It is, however unavoidable that, at times, the fading is severe and results in a small value for γ_I . This can cause a drastic degradation in the performance. To provide robustness under such circumstances, we resort to transmission energy allocation.

To estimate the distortion resulting from a severe fading condition, we adopt the following approach. Let γ_{rep} denote the representative value taken by γ_I under severe fading. Neglecting more than one bit errors per index, the net representative channel distortion (not including the distortion due to quantization) for the I -th user is

$$D_I = \sum_i \beta_{Ii} \operatorname{erfc} \left(\sqrt{\frac{\gamma_{rep}^2 \alpha_I^2 E_{Ii} C}{\sum_{J \neq I} \alpha_J^2 E_{Ji}}} \right). \quad (5)$$

The representative distortion averaged over the users is, therefore, $D = \frac{1}{M} \sum_{I=1}^M D_I$. Clearly, this average distortion depends on:

1. The distribution of transmission energy among the bits of the various users.
2. The order of transmission of the k bits by the different users.

We propose to minimize the average distortion D , by appropriate allocation of transmission energy *and* ordering of bits for each user. As in the power control problem, we impose the constraint that the total energy transmitted by all the users in any bit interval is constant:

$$\sum_I E_{Ii} = E_{tot}, \quad \text{for all } i. \quad (6)$$

As mentioned earlier, we are interested in imparting robustness to VQ index transmission for each user I , when its rapidly varying gain γ_I , happens to take low values. Later, we show that, by appropriately choosing the value of γ_{rep} , we can significantly enhance the robustness of the VQ over a range of gain values, γ_I .

4 Proposed Solution

We begin by defining the average representative distortion incurred in the i -th bit duration, $D(i)$ as

$$D(i) = \frac{1}{M} \sum_I \beta_{Ii} \operatorname{erfc} \left(\sqrt{\frac{\gamma_{rep}^2 \alpha_I^2 E_{Ii} C}{\sum_{J \neq I} \alpha_J^2 E_{Ji}}} \right). \quad (7)$$

The overall average representative distortion, D , can be written in terms of these bit-duration distortions as $D = \sum_i D(i)$. It is important to note that, (i) the constraint (6) on the transmission energy values $\{E_{Ii}\}$, is imposed in terms of the total energy transmitted in each bit interval, and (ii) the values $\{E_{Ii}\}$ do not affect the value of distortion $D(j)$, in bit intervals $j \neq i$. This implies that to minimize the overall distortion, it is sufficient to minimize the distortion $D(i)$ over each bit interval. Hence for a given ordering of bits of the different users, our optimization problem can be decoupled into the following k sub-problems:

For $i = 1$ to k , allocate transmission energies $\{E_{Ii}\}$ satisfying the constraint (6) to minimize $D(i)$ given by (7).

Before presenting the solution to the general problem, it is helpful to consider the special case where all the user signals experience the same channel gain. The solution to the general problem can be easily obtained from the solution to this special case.

4.1 Solution to the Special Case

Consider the case when all the user signals experience the same level of slowly varying channel gain. In other words, we have $\alpha_I = \alpha$, for all I . For this case we can easily show that, for a given ordering of source bits, the optimal solution will allocate the same amount of transmission energy to all the users transmitting the bit of the same sensitivity in a particular bit interval. During the optimization, it is useful to explicitly impose this constraint on the transmission energy values. Specifically, if we denote by e_{ij} the transmission energy allocated to a bit of sensitivity β_j in the i -th bit interval, and if we denote by m_{ij} the number of users transmitting a bit of sensitivity β_j in the i -th bit interval, then the overall representative distortion can be simplified to

$$D(i) = \sum_j m_{ij} \beta_j \operatorname{erfc} \left(\sqrt{\frac{\gamma_{rep}^2 C e_{ij}}{E_{tot} - e_{ij}}} \right). \quad (8)$$

The constraint on the transmission energies can be rewritten as $\sum_j m_{ij} e_{ij} = E_{tot}$.

The solution to this problem involves choosing an appropriate ordering of bits and optimizing the transmission energy values for this bit ordering. Since the solution depends on the actual bit ordering only via the values $\{m_{ij}\}$, it is worthwhile to take a closer look at these quantities.

It is easy to see that the quantities $\{m_{ij}\}$ take (non-negative) integer values and satisfy the following equations

$$\sum_j m_{ij} = M, \quad \text{for } \forall i, \text{ and}$$

$$\sum_i m_{ij} = M, \quad \text{for } \forall j. \quad (9)$$

Let us combine the set of quantities $\{m_{ij}\}$ into a matrix \mathcal{M} . We will refer to this matrix as the bit enumerator matrix. Clearly many bit orderings will result in the same \mathcal{M} matrix and hence the same solution for the transmission energy allocation problem. We denote this solution by $E^* = \{e_{ij}^*\}$ and the corresponding distortion by $D^*(\mathcal{M})$.

Directly searching for an ordering of bits, such that the resulting $D^*(\mathcal{M})$ is minimized is a formidable task. Instead we propose to search for the set of non-negative, real values m_{ij} which satisfy (9) and minimize the overall distortion. Clearly, we are looking for solutions in a space much larger than that allowed by the ordering of bits. We will, however, show that the optimal set either corresponds to, or can be closely approximated via an ordering of bits.

In developing a solution to this problem we will use the following two properties of the optimal solution:

Property 1: $D^*(\mathcal{M})$ is invariant to row switching in \mathcal{M} . This is due to symmetry, as such switches are simple renaming of bit intervals.

Property 2: If $e_{ij}^* \leq \frac{E_{tot}}{4}$, it can be shown that $D^*(\mathcal{M})$ is a convex function. (see Appendix).

With any realistic number of users, almost always, we will have transmission energy allocated to any bit is less than $\frac{E_{tot}}{4}$. Hence we can assume that $D^*(\mathcal{M})$ is a convex function in our region of interest. Thus we are minimizing a convex function over the set defined by the constraints, (9). These constraints themselves define a convex set. It follows that there is a unique minimizer which we denote by \mathcal{M}^* . However, property 1 implies that we may obtain equivalent minimizers by switching rows in \mathcal{M}^* . The only solution satisfying both properties and (9) is $m_{ij} = \frac{M}{k}$.

A bit-ordering which realizes (or approximates) this solution can be obtained as follows. We start with any ordering of bits for the user 1. Now we cyclically shift the bits of user 1 and use this ordering of bits for user 2. We continue in this manner, with user- i ordering its bits in a pattern obtained by cyclically shifting the bit ordering pattern of the user $i - 1$. Clearly, if M is a multiple of k , we implement the exact optimal solution. If M is not a multiple of k , we implement an approximation to the optimal solution.

Remark: When we implement the optimal solution, $m_{ij} = \frac{M}{k}$, the level of transmission energy e_{ij} , is only a function of β_j , i.e. $e_{ij} = e(\beta_j)$. Thus the total transmission energy allocated to each user $\sum_j e(\beta_j)$, is a constant. Note that we posed the problem so that the total energy transmitted by the users per bit interval is constant. The optimal solution, however, assigns equal amount of net transmission energy to each user. Moreover, this transmission is distributed among the different bits according to their sensitivity. This solution is very similar to the solution of TEA in a noise-limited environment, as described in section 2.

4.2 General Case

We now address the general case when $\{\alpha_I\}$ can take any values. We will first show that by minimizing the distortion subject to any linear constraint on the transmission energy values, the same value for distortion is obtained. We then show that by choosing a particular constraint, the optimization problem reduces to the special case described in the last subsection.

Claim: The minimum of $D(i)$ over the transmission energy allocation $\{E_{Ii}\}$, subject to linear constraint $\sum_I \theta_I E_{Ii} = E'_{tot}$, is invariant to the choice of coefficients $\{\theta_I\}$.

Proof: Let the set of values $\{\theta_{1I}\}$ and $\{\theta_{2I}\}$ result in minimum distortion values $D1(i)$ and $D2(i)$ respectively, with $D2(i) \leq D1(i)$. Let us denote the corresponding optimal transmission energy values by $\{E1_{Ii}\}$ and $\{E2_{Ii}\}$. Consider scaling the energy values, $\{E2_{Ii}\}$, by a positive real number, δ , such that the resulting transmission energy values satisfy $\delta \sum_I \theta_{1I} E2_{Ii} = E'_{tot}$. It is easily seen that such a scaling keeps the value of distortion unchanged. Thus the set $\{\delta E2_{Ii}\}$ satisfies the constraint $\sum_I \theta_{1I} E_{Ii} = E'_{tot}$, and achieves distortion $D2(i)$ which is smaller than or equal to $D1(i)$. However, we know that the transmission energy values $\{E1_{Ii}\}$, minimize the distortion, $D(i)$, subject to constraint $\sum_I \theta_{1I} E_{Ii} = E'_{tot}$. Hence we should have $D1(i) = D2(i)$.

The specific constraint that we wish to impose on the transmission energy values while minimizing $D(i)$ is $\sum_I \alpha_I^2 E_{Ii} = E'_{tot}$. In other words, we impose the constraint that the received signal energy from all the users in each bit interval is a constant. Let the solution to this problem be denoted by $\{E'_{Ii}\}$. As a consequence of the result proved above, by scaling these transmission energy values by a factor δ_i , such that $\sum_I \delta_i E'_{Ii} = E'_{tot}$, we will have solved the optimization problem of our interest.

We can simplify the new optimization problem by rewriting it as follows. We denote the product $\alpha_I^2 E_{Ii}$ by \tilde{E}_{Ii} . Thus we would like to minimize

$$D(i) = \sum_I \beta_{Ii} \operatorname{erfc} \left(\frac{\gamma_{ref}^2 \tilde{E}_{Ii} C}{E'_{tot} - \tilde{E}_{Ii}} \right)$$

subject to the constraint $\sum_I \tilde{E}_{Ii} = E'_{tot}$. This is precisely the problem solved in the last subsection, and the solution is therefore to have $\frac{M}{k}$ users transmit bits of each sensitivity in every bit interval and optimize the values of \tilde{E}_{Ii} . Subsequently, we scale the corresponding transmission energy values such that the total energy transmitted in each modulation interval is E'_{tot} .

Remarks:

(1) If all the source bits are equally sensitive to channel errors, we obtain the solution $\tilde{E}_{Ii} = \text{const}$, that is $\alpha_I^2 E_{Ii} = \text{const}$. Hence we allocate power to the users, such that the received signal power for each bit is constant. This is the standard power control strategy. In other words, the standard power control solution is included as a special case of this scheme.

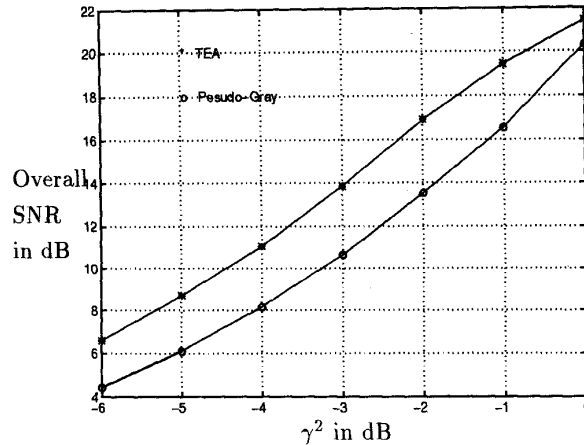


Figure 1: The overall SNR versus the level of rapidly varying channel gain, γ^2 . The source is a Gauss-Markov with correlation coefficient 0.8. This source is quantized via a 8-bit, 2-dimensional VQ and transmitted over a Nakagami-3 fading channel using CDMA.

(2) If the optimal bit ordering that yields $m_{ij} = \frac{M}{k}$ is used, we have $\sum_i \tilde{E}_{Ii} = \text{constant}$ for all users (recall the remark at the end of last subsection). We can explicitly write this as $\sum_i E_{Ii} = \frac{\text{constant}}{\alpha_I^2}$. That is, the total transmission energy received from each user is kept constant. Unequal error protection is achieved by redistributing this energy among the k bits according to their sensitivity. That is, we first allocate the total transmission energy per codevector to each user via the standard power control approach, which is then only redistributed among the different bits to yield the desired unequal error protection.

5 Results

We demonstrate the performance advantage of the proposed TEA scheme over the standard pseudo-Gray coding scheme in a fading environment.

We consider a multi-user system consisting of 40 users transmitting a coded source using CDMA. The source-coder we employ is a vector quantizer with a bit rate of 8 bits per source vector. In terms of the notation defined earlier, $M = 40$ and $k = 8$. Since M is a multiple of k , we can implement the optimal TEA solution, wherein exactly $\frac{M}{k} = 5$, users transmit bits of each sensitivity within each bit transmission interval. The value of the gains $\{\alpha_I\}$, for each of the users were taken to be uniformly distributed between 0.01 and 1.0. The values of the rapidly varying gain factors γ_I , were taken to be according to Nakagami-3 distribution. That is, we assume that three multipath signals, with Rayleigh fading on each, have been optimally combined to yield the received signal. To set the value of the parameter, C - the number of chips per bit, we proceed as follows. We assume that the value of $\gamma_I = 1$, and that the allocation of transmission energy per bit to each user is

such that $E_{Ji}\alpha_I = \text{const}$. Under such conditions, the value of the signal to interference ratio can be shown to be $\frac{C}{M-1}$. We set C such that a bit error rate of less than 10^{-3} is obtained. The corresponding value of C is 380.

The TEA scheme is designed as described in section 4. We compare performance with the standard scheme which employs a VQ with pseudo-Gray code based indexing [8]. The pseudo-Gray scheme allocates equal transmission energy to the bits and its value is determined by the power control scheme which ensures that $\alpha_i^2 E_{Ii} = \text{const}$. We evaluate the performance of each system as follows. Let x denote the source vector to be quantized. The corresponding reproduction vector at the receiver will be denoted by \hat{x} . We consider the distortion incurred for a particular user, I , when its rapidly varying channel gain is $\gamma_I = \gamma$. At this instant, other users have their rapidly varying channel gains distributed as Nakagami-3 variables. We can estimate the distortion for the I -th user by simulation. We now consider similar scenarios with respect to each user, that is, the particular user J , has $\gamma_J = \gamma$ and other users have their rapidly time-varying attenuation factors Nakagami-3 distributed. The resulting distortion averaged over all the users is

$$D(\gamma) = \frac{1}{M} \sum_I E [\|x - \hat{x}\|^2 | \epsilon_{1i}, \epsilon_{2i}, \dots, \epsilon_{ki}], \quad (10)$$

where ϵ_{Ii} is the bit error rate for the i -th bit of the user- I and is given by

$$\epsilon_{Ii} = \text{erfc} \left(\sqrt{\frac{\gamma^2 \alpha_i^2 E_{Ii}}{\sum_{J \neq I} \alpha_j^2 E_{Ji}}} \right). \quad (11)$$

In figure 1, we compare the performance of the TEA and pseudo-Gray schemes for the case of a Gauss-Markov source with correlation coefficient 0.8, blocked into vectors of size 2. The TEA scheme was optimized for a value of $\gamma_{rep}^2 = \sqrt{0.5}$. The performance is evaluated for values of γ^2 in the range of -6 to 0 dB. As can be seen, the TEA scheme obtains substantial performance gains of the order of 2-3 dB over the standard pseudo-Gray coding scheme.

6 Conclusions

Transmission energy allocation (TEA) is a promising technique that substantially improves the robustness of a vector quantizer when the transmission channel is time-varying. In this work we addressed the problem to applying the technique of TEA in a multi-user environment. In particular we considered the case of a CDMA with multi-user interference from only within the cell. Important issues addressed in this context included: the effect of TEA on the level of multi-user interference, ordering of bits of the different users and the interaction between TEA and the power control scheme. We developed a framework to address these problems jointly. The resulting solution turns out to be simple and easily implementable. A comparison of the TEA

scheme over the standard pseudo-Gray coding scheme with equal transmission energy allocated to each source bit was presented for the case of Gauss-Markov source. It was shown that, especially under conditions of severe fading, TEA obtains large performance gains, of the order of 2-3 decibels over the pseudo-Gray coding scheme.

Appendix

Claim: $D^*(M)$ is a convex function, as long as the transmission energy allocated to each bit is less than $\frac{E_{tot}}{4}$.

Proof: Let $\mathcal{M}_1 = \{m_{1ij}\}$ and $\mathcal{M}_2 = \{m_{2ij}\}$ be two, real bit enumerator matrices, with the corresponding optimal transmission energy allocations $E_1^* = \{e_{1ij}^*\}$ and $E_2^* = \{e_{2ij}^*\}$ respectively. Consider $\mathcal{M}' = \lambda \mathcal{M}_1 + (1 - \lambda) \mathcal{M}_2$, and a particular set of transmission energy values given by $E' = \{e'_{ij}\}$, with $e'_{ij} = \frac{\lambda m_{1ij} e_{1ij}^* + (1 - \lambda) m_{2ij} e_{2ij}^*}{\lambda m_{1ij} + (1 - \lambda) m_{2ij}}$. It is easy to see that \mathcal{M}' and $E' = \{e'_{ij}\}$ satisfy the constraints on total transmission energy per bit interval. The resulting distortion is $D(\mathcal{M}', E')$. It can be shown that $\text{erfc} \left(\sqrt{\frac{\gamma_{rep}^2 C x}{E_{tot} - x}} \right)$ is a strictly convex \cup function of x , for $x < \frac{E_{tot}}{4}$. Assuming that E_1^* and E_2^* consist of elements with value less than $\frac{E_{tot}}{4}$, we can show that

$$D(\mathcal{M}', E') < \lambda D^*(\mathcal{M}_1) + (1 - \lambda) D^*(\mathcal{M}_2).$$

The optimal $D^*(\mathcal{M}')$ will be less than or equal to $D(\mathcal{M}', E')$. Hence we have

$$D^*(\mathcal{M}') < \lambda D^*(\mathcal{M}_1) + (1 - \lambda) D^*(\mathcal{M}_2).$$

That is, $D^*(\mathcal{M})$ is a convex function.

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