Unequally Protected Multi-Stage Vector Quantization for Time-Varying Channels

Shrinivas Gadkari and Kenneth Rose
Department of Electrical and Computer Engineering
University of California, Santa Barbara, CA 93106

Abstract—**We present a source-channel coding system consisting of a multi-stage vector quantizer in conjunction with unequal protection against channel errors. The receiver estimates the channel conditions and decodes as many stages of the quantized signal as can be reliably decoded. The proposed overall system is optimized for robustness to time-varying noisy channels. Two versions of the system are designed and evaluated, which involve: (i) hard, and (ii) weighted decoding of the stage indices. Simulation results are given for Gauss-Markov sources transmitted over broadcast and fading channels. Consistent and substantial improvement is achieved over the standard multi-stage vector quantizer with equal error protection, and the gains are in the range of 3 to 5 dB.**

I. INTRODUCTION

In this work, we address the problem of designing a joint source-channel coding system for operation over time-varying channels. We make the following assumptions: (i) The transmitter has no access to precise channel condition information, but has knowledge of a priori statistical description of the channel condition (e.g., probability distribution of the level of attenuation in the channel). (ii) The receiver has access to information on the current state of the channel and uses it during the decoding process. Communication scenarios which motivate this problem with the above assumptions include: (i) broadcast channels, and (ii) mobile communication channels without feedback path from receiver to transmitter.

In the case of broadcast applications, a single transmitter transmits an encoded signal to many receivers. Depending on its location and equipment, each receiver experiences a different channel condition in terms of the received signal strength and the level of interference (and noise) power.

Next consider a mobile communication scenario. Here, the information about the time-varying channel characteristics is often available at the receiver, and can be used during the decoding process. However, if a feedback path is not available (or is not feasible), the transmitter has no access to this information.

Clearly, both these scenarios are very similar. An appropriate source-channel coding strategy for these applications was suggested by Cover in his classic paper on broadcast channels [2]. It consists of employing a multi-resolution source coder followed by unequal protection of its different bit streams. The coarse level bit stream is given the heaviest protection while the finer resolution bit streams are given lighter protection. Thus, the coarse level bit stream can be decoded even when the channel conditions are relatively poor providing a lower quality reproduced signal. When the channel conditions improve, the finer resolution bit streams can also be decoded resulting in a higher fidelity reproduced signal. Among the first papers to describe the design of a practical source-channel coder based on this philosophy is the work of Ramchandran et al [1], where they consider the design of a multi-resolution image broadcast scheme.

In this work, we consider a system consisting of a multi-stage vector quantizer (MSVQ) followed by unequal protection of the different stages. MSVQ is a widely used structured vector quantizer, most notably in speech coding. MSVQ performs successively refined quantization of the source vector, where the early stages produce a coarse approximation of the source signal, and the later stages provide the finer details. The early stages are given heavy protection to ensure decodability even under poor channel conditions. The later stages, which are given lighter protection, are decoded when the channel is cleaner, and produce enhanced signal quality. For the sake of simplicity, we specialize the presentation to unequal protection methods which are appropriate for the CDMA scenario. However, the ideas are more generally applicable and can be easily extended to other spectrum sharing methods like TDMA and FDMA.

II. THE SYSTEM

The scheme we consider consists of two main components: unequal error protection (UEP) and multi-stage vector quantizer (MSVQ). Here we briefly describe the two components separately, and then combine them to formulate the overall system design problem.

A. Unequal Error Protection

Consider a UEP scheme where several bit streams $B_1, B_2, ..., B_k$ need to be protected against different levels of channel noise. Let R_i denote the rate of B_i in bits per second, and N_i be the level of noise that B_i needs to withstand. In other words, B_i should be decodable with

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bit error rate below a prespecified acceptable value, as long as the level of channel noise, N, does not exceed N_i, i.e. $N \leq N_i$. Whenever necessary for obtaining numerical results, we will assume that bit error rate of 10^{-3} is "acceptable".

The choice of UEP technique depends on the multiple access method employed to share the available radio spectrum among the different users. In the case of CDMA, where each user occupies the entire spectrum, a simple unequal error protection scheme can be devised as follows. We first encode each bit stream via a suitable, binary channel code, for example, a rate 1/2 convolutional code. For bit stream B_i, the channel code produces a sequence of binary symbols $\{v_i(j)\}$, where $v_i(j) \in \{+1, -1\}$. We provide unequal protection by transmitting the symbols corresponding to different bit streams with different values of transmission energy. Let e_i denote the *energy per information bit* employed for transmission of bit stream B_i. Since we consider employing a rate 1/2 code, each channel symbol $v_i(j)$, is transmitted with energy $\frac{e_i}{2}$. The corresponding modulated signals $\{V_i(t)\}$, are spread using orthonormal pseudo-noise waveforms, $\{PN_i(t)\}$. The resulting spread signal $s(t) = \sum_{i=1}^{k} V_i(t)\, PN_i(t)$, is then transmitted on the channel.

At the decoding end, we receive $r(t) = s(t) + n(t)$, where, $n(t)$ is Gaussian channel noise of variance N, whose value varies with time.[1] To decode the i-th bit stream of the particular user under consideration, we first despread $r(t)$ using the pseudo-noise waveform $PN_i(t)$ and then recover B_i.

The level of protection provided to each bit stream can be determined as follows: Let γ^* denote the minimum value of the received channel SNR which ensures that the decoded bit error rate is below the "acceptable" value. It is easy to see that the value of the received channel SNR per bit for bit stream B_i is related to its transmission energy value e_i and to the variance of the channel noise N, through $\frac{e_i}{N}$. Thus, the maximum level of channel noise variance that the bit stream B_i can withstand, is given by, $N_i = \frac{e_i}{\gamma^*}$.

B. Multi-Stage VQ - Basic Structure

An M-stage MSVQ consists of M codebooks, C_1, C_2, .., C_M. The codebook C_i is a set of 2^{r_i} codevectors addressable by an r_i bit index, I_i. A given source vector x, is approximated by

$$\hat{x} = u_1(I_1) + u_2(I_2) + \ldots + u_M(I_M), \qquad (1)$$

where, $u_i(I_i)$ is the codevector in C_i, which is indexed by I_i. The objective of the encoding operation is to select a codevector from each codebook such that the error $d(x, \hat{x})$ is minimized, where $d(\cdot,\cdot)$ is a distortion measure. The set of indices $(I_1, I_2, \ldots, I_M)$ is transmitted and allows the decoder to produce $\hat{x}$. Ideally, we would like to perform an exhaustive search to find the best combination of indices $(I_1, I_2, \ldots, I_M)$ to be transmitted. However, this computation can be prohibitively complex, and instead one may adopt a procedure called M-L search that approximates the optimal exhaustive search, (see e.g. [3]).

M-L Search: We first compare the source vector x with all the code vectors in C_1 and select the L vectors which best approximate x in terms of minimizing the distortion $d(\cdot,\cdot)$. These are the L survivors at stage 1 and will be denoted by $\{u^{(l)}{}_1, l = 1, .., L\}$. In the next step, we consider all possible combinations of each of the L survivors with the codevectors in C_2 and choose the L combinations $\{u^{(l)}{}_2\}$ that best approximate x. We now have L survivors at stage 2. We repeat this procedure of selecting L survivors until we reach the M-th stage where we select the best approximation among the L survivors, and this determines $\hat{x}$ and concludes the encoding operation.

C. System Description

We consider an M-stage MSVQ with 2^{r_i} codevectors in the i-th stage. The values of $\{r_i\}$ are chosen such that the size of each stage-codebook is manageable. Each stage-index is protected unequally by allocating a different value of energy per bit for its transmission via the scheme described in section IIA. Specifically, let e_i be the energy per bit allocated for transmission of the i-th stage index. The minimum received channel SNR per bit needed to decode an index with bit error rate below the prescribed value is denoted by γ^*. Thus the i-th stage-index is protected against noise of level N_i, which is related to e_i via, $N_i = \frac{e_i}{\gamma^*}$. At the receiving end, we estimate the current level of channel noise N, and decode as many stage-indices as can be reliably decoded. Thus if $N_{i+1} < N \leq N_i$, the indices $I_1, I_2, \ldots, I_i$ are decoded. The corresponding reproduction is given by

$$\hat{x} = u_1(I_1) + \ldots + u_i(I_i).$$

The resulting distortion is measured as the squared error, $d(x, \hat{x}) = \| x - \hat{x} \|^2$. Note that as the level of channel noise N, decreases, a larger number of stage indices can be decoded. Consequently, a smaller distortion is incurred.

The problem that we attack is that of designing the overall system to minimize the average distortion, $\overline{D} = E\left[\| x - \hat{x} \|^2\right]$, while meeting the given constraints on the total transmission energy per source vector: $\sum_i r_i\, e_i = E_{tot}$.

We assume, as explained earlier, that statistical knowledge of the level of channel noise N, is available in the form of the probability density function $p(n)$, and can be used during the system design. The system design, consists of:

1. Determination of an appropriate codebook search procedure.
2. Design of the decoder codebooks, and
3. Optimization of the UEP scheme, via suitable allocation of the available transmission energy.

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[1] In most of the mobile communication channels, it is the level of received power that varies with time, while the variations in the level of channel noise are minimal. However, for the current discussion, we find it more convenient to consider instead the equivalent setup where the received signal power is constant and the level of channel noise varies.

III. SYSTEM DESIGN

A. Encoding Search

The objective of the encoder is to determine the best set of indices $(I_1, I_2, ..., I_M)$ to be transmitted over the channel. The information accessible to the encoder includes the set of decoder codebooks $\{C_i\}$, the unequal protection levels N_i, and the probability density function of level (power) of channel noise, $p(n)$.

The conditional expected value of the distortion, given the transmission of indices $I_1, I_2, ..., I_M$, is given by

$$D\left(x|I_1, I_2, ..., I_M\right) = \sum_{i=1}^{M} P_i \parallel x - \sum_{l=1}^{i} u_l(I_l) \parallel^2, \qquad (2)$$

where P_i denotes the probability that the decoding *stops* at the i-th stage, we have $P_i = \int_{N_{i+1}}^{N_i} p(n)\, \mathrm{d}n$, where we set $N_{M+1} = 0$. We refer to $\{P_i\}$ as the "stage decoding probabilities".

Modified M-L Search: The objective here is to approximate the exhaustive search for the set of indices $(I_1, I_2, ..., I_M)$ which minimize the distortion (2). As described in section IIB, the basic idea is to proceed sequentially from stage-1 to stage-M, retaining L survivors at each stage. In principle, one wishes to minimize the distortion (2) at each stage. However, at stage J we do not have information about vector selection of the subsequent stages. We therefore approximate all subsequent stage codevectors by 0 (in general one would use their mean value but in the case of MSVQ the mean is approximately 0). Hence, the encoding cost function at the J th stage is

$$D_J(x) = \sum_{i=1}^{M} P_i \parallel x - \sum_{l=1}^{i} u_l(I_l) \parallel^2, \text{ where } u_l = 0 \text{ for } l > J. \qquad (3)$$

Using this cost function, we perform an M-L search. The ultimate survivor determines the selected codevectors $\{u_i\}$. The corresponding set of stage-indices is transmitted over the channel.

B. Iterative Design

To optimize the decoder codebooks and the UEP scheme, we employ an iterative design method where each iteration consists of two complementary steps, namely, codebook optimization (MSVQ design), and transmission energy allocation (UEP design). The following is a high level summary of the overall algorithm:

1. Choose an initial set of MSVQ codebooks and an initial UEP energy allocation scheme.
2. Fix the energy allocation and reoptimize the codebooks (MSVQ design).
3. Fix the codebooks and reoptimize the energy allocation (UEP design).
4. Check for convergence to stop.
5. Go to (2).

In the next two subsections we describe in detail, steps (2) and (3) - the two principal steps of the iteration.

B.1 MSVQ Design for a given Transmission Energy Allocation

Given a training set $\mathcal{T} = \{x\}$, protection levels for the different stages $\{N_i\}$, and the decoder codebooks obtained from the design of the previous iteration (or from the initialization if this is the first iteration), we reoptimize the decoder codebooks as follows. In each cycle of the iteration, one codebook is optimized. We denote the index of this codebook by i.

1. Set $i = 1$.
2. Encode the training set $\mathcal{T}$ and partition it into subsets $\{R_I\}$, $I = 1, 2, ..., 2^{r_i}$, where, R_I consists of training vectors in $\mathcal{T}$ to which the search procedure assigns, I, as the i-th stage index.
3. Adjust the entries of the codebook C_i, to minimize
$$D = \sum_{x \in \mathcal{T}} \left\{ \sum_{j=1}^{M} P_j \parallel x - \sum_{k=1}^{j} u_k \parallel^2 \right\}, \qquad (4)$$
while keeping the codebooks $\{C_l\}_{l \neq i}$ fixed.
4. If $i < M$, increment i, and go to step 2. If $i = M$, check for a stopping criterion. If not converged, go to step 1.

B.2 Optimizing the UEP Scheme for a given MSVQ

Given an MSVQ, we need to design the UEP scheme which provides protection for its various stages. In other words, we need to find the set of transmission energy values $\{e_i\}$, satisfying $\sum_i r_i\, e_i = E_{tot}$, such that the average distortion is minimized.

Although one may conceive of sophisticated methods to optimize the UEP, the low complexity of the problem allows a very simple approach:

1. Generate several candidate energy allocations $\{e_i\}$, that meet the constraint on the total transmission energy.
2. Evaluate the average distortion $\overline{D}$ for each set.
3. Choose the energy allocation that minimizes the average distortion.

Evaluation of $\overline{D}$ for a set of energy allocation $\{e_i\}$ values is a low complexity operation and can be performed as described next. Let D_i denote the average distortion incurred when the decoding is stopped at the i-th stage. That is, $D_i = \sum_{x \in \mathcal{T}} \parallel x - \sum_{j=1}^{i} u_j \parallel^2$. For each set of energy allocation values, we first evaluate the stage decoding probabilities $\{P_i\}$. The average distortion $\overline{D}$ can now be evaluated using the values $\{D_i\}$ and $\{P_i\}$ as $\overline{D} = \sum_{i=1}^{M} P_i\, D_i + \left(1 - \sum_{i=1}^{M} P_i\right) E \parallel x \parallel^2$, where, the last term accounts for the fact that, with probability $1 - \sum_{i=1}^{M} P_i$, none of the stage indices are decoded.

IV. RESULTS

We designed a 4-stage MSVQ to quantize a Gauss-Markov source with correlation coefficient 0.8 which was blocked into 6-tuples. Each MSVQ stage generated a 6-bit index, which was encoded using a rate 1/2, constraint

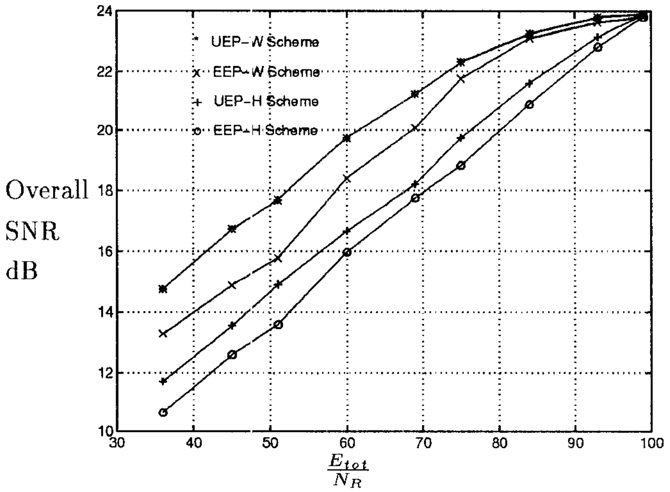

Fig. 1. A comparison of the performance of different MSVQ schemes in a broadcast scenario. UEP-H: UEP scheme with hard decoding, UEP-W: UEP scheme with weighted decoding, EEP-H: EEP scheme with hard decoding, EEP-W: EEP scheme with weighted decoding.

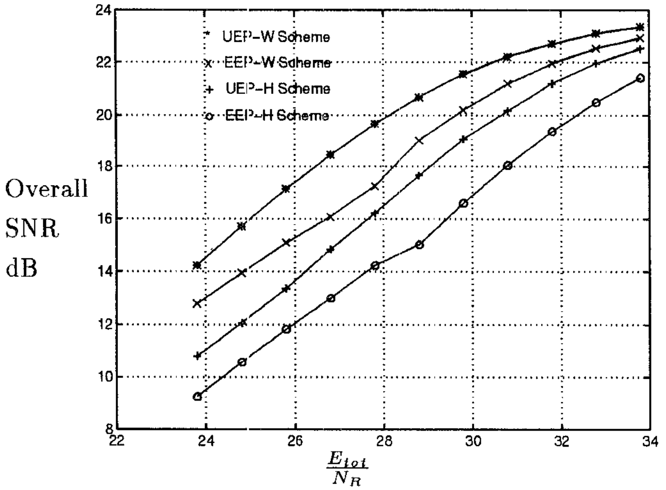

Fig. 2. A comparison of the performance of different MSVQ schemes over a Nakagami-2 fading channel. UEP-H: UEP scheme with hard decoding, UEP-W: UEP scheme with weighted decoding, EEP-H: EEP scheme with hard decoding, EEP-W: EEP scheme with weighted decoding.

length-6, convolutional code. For such a transmission scheme, a stage-index can be decoded with bit error rate below 10^{-3} as long as the received channel SNR per (index) bit is above 6 dB, that is $\gamma^* = 4.0$. Unequal protection was provided to the different stage indices via appropriate allocation of transmission energy. We will refer to this system as the UEP-MSVQ scheme.

The performance of the UEP-MSVQ scheme was compared to that of an equal protection MSVQ (EEP-MSVQ) scheme which distributes the transmission energy uniformly among all the stage indices. It is important to emphasize that in the case of EEP-MSVQ, for a given value of total transmission energy E_{tot}, one has the freedom to decide on the optimal number of bits to be employed for quantization. To ensure fairness of comparison, we considered the performance of several MSVQ schemes employing overall bits in the range of 6 to 24, with equal transmission energy per bit allocated to each stage index. The scheme that yielded the best performance was chosen to represent the EEP-MSVQ in the results.

For both UEP-MSVQ and EEP-MSVQ systems, a value of $M = 10$ was used during the M-L search.

A. Broadcast Channel

Let us consider a broadcast scenario where a base station is transmitting a coded signal to several receivers within a cell of radius R. The strength (power) of the received signal, at distance r from the base station is modeled by $E(r) \propto \frac{1}{r^k}$. The value of k depends on the characteristics of the region. We will assume $k = 4$, a typical value for cellular environments [4]. For simplicity, we further assume that the users are uniformly distributed in the cell, and that the total noise (plus interference) power is N_0. The pdf of the received channel SNR can now be estimated. As mentioned earlier, for convenience we consider the equivalent situation where the received signal power is constant (independent of the distance from the base station), but the noise power varies. It can be easily shown that the pdf of this effective channel noise is given by $p(n) = \frac{0.5}{\sqrt{N_R n}}$, for $0 < n \leq N_R$, where, N_R is the noise power at distance R from the base.

In Figure 1, the performance of the UEP-MSVQ scheme is compared to that of the EEP-MSVQ scheme for different values of $\frac{E_{tot}}{N_R}$ (the performance of the UEP-MSVQ and the EEP-MSVQ is depicted by the UEP-H and EEP-H curves respectively). The plots show that UEP-MSVQ achieves performance gains of about 1 dB over EEP-MSVQ. Note also that the gains are more pronounced at small values of $\frac{E_{tot}}{N_R}$.

B. Fading Channel

Here we compare the performance of UEP-MSVQ and EEP-MSVQ over a Nakagami-m fading channel. A Nakagami-m fading model applies when we have an m-path diversity, with independent Rayleigh fading on each of the paths. In particular, we consider the case of $m = 2$. For this channel the received channel SNR (γ) has a pdf $p(\gamma) = \frac{4\gamma}{\bar{\gamma}^2} \exp\left(-\frac{2\gamma}{\bar{\gamma}}\right)$, where, $\bar{\gamma}$ is the average received channel SNR. Again we consider an equivalent scenario where the received signal power is fixed and the noise power varies, with average noise power equal to $\bar{n}$. The corresponding pdf can be estimated numerically. The performance of the UEP-MSVQ and EEP-MSVQ schemes for various values of $\frac{E_{tot}}{\bar{n}}$ is depicted in Figure 2, where they are labeled UEP-H and EEP-H, respectively. We observe that, in this case, large performance gains of about 2 dB can be achieved by the UEP scheme.

V. WEIGHTED DECODING OF STAGE INDICES

So far we assumed that the stage index was either completely decoded (if the current level of channel noise was below the protection level) or not decoded at all. We, naturally, refer to this as hard decoding. In this section we explore the possible advantages of employing a weighted decoding rule. Here, instead of completely rejecting unreliable indices, we decode them while taking into account their degree of reliability.

Consider the following scheme where we decode *all* the stage indices irrespective of the level of channel noise. For the i th stage we have been denoting the transmitted index by I_i. We denote by J_i, the corresponding received index. Since the decoder knows the current level of channel noise, and also the transmission energy values employed for the transmission of stage-indices, it has an estimate of the error rate in decoding J_i. The reconstruction rule that we propose to use is given by

$$\hat{x} = \hat{u}_1(J_1) + \hat{u}_2(J_2) + \ldots + \hat{u}_M(J_M), \qquad (5)$$

where, $\hat{u}_i$ is the optimal estimate of the i-th stage codevector given the received stage index J_i and the current level of channel noise N:

$$\hat{u}_i(J_i) = E\left[u_i | J_i, N\right] = \sum_{I_i} P(I_i | J_i, N)\, u_i(I_i). \qquad (6)$$

Observe that if N is below N_i, then the error rate for stage i is low and $\hat{u}_i$ given by (6) approximates $u_i(J_i)$. On the other hand, if $N >> N_i$, $\hat{u}_i \approx 0$ (or more generally, the mean of the codebook C_i). Thus for these cases, the weighted decoding rule simplifies to the hard decoding rule. However when N is close to N_i, the weighted decoding rule (for the i th stage) significantly differs from the hard decoding rule.

Complexity of Weighted Decoding: The computational complexity of estimating $\hat{u}_i$ is proportional to the size of the codebook, C_i, that is 2^{r_i}. However, we know that the size 2^{r_i} is small enough to allow manageable encoding search complexity. Hence the computation of $\hat{u}_i$ using (6) should be feasible. We conclude that weighted decoding is implementable in most practical systems.

System Design for Weighted Decoding: The design of the UEP scheme with hard decoding was described in section IIIB.2. The key idea there was to evaluate the average distortion ($\overline{D}$) for several energy allocations, and choose the energy allocation that minimizes the average distortion. With weighted decoding we can employ the same optimization strategy with the following modification in evaluation of the average distortion:
$\overline{D} = \int E \parallel x - \sum_{i=1}^{M} \hat{u}_i(n) \parallel^2 \, p(n)\, dn$.

Performance of Weighted Decoding: We implemented the weighted decoding of stage indices for the design examples described in section IV. The results for the broadcast channel are included in Figure 1, and the results for the Nakagami-2 fading channel are illustrated in Figure 2 (depicted by the curves labeled UEP-W).

It is, of course, also possible to employ weighted decoding with equal error protection. We implemented and evaluated the performance of such a system for the case of broadcast and fading channel examples. These results are included in the corresponding figures, labeled as EEP-W.

We make the following observations:

1. The UEP-W scheme achieves large performance gains in the range of 2-3 dB over the UEP-H scheme for both broadcast and fading channel examples.
2. As a consequence of weighted decoding, the performance of the equal error protection scheme also improves substantially (by over 2 dB).
3. The overall performance gains of the UEP-W scheme over the standard EEP-H scheme, for low to moderately high values of E_{tot}, are in the rough range of 3-5 dB.

These results clearly emphasize the importance of both unequal protection and weighted decoding of the stage indices. It should be re-emphasized that, both these features can be implemented with feasible overall complexity.

VI. CONCLUSIONS

This work proposes a new approach to the problem of designing a multi-resolution (successively refined) vector quantization scheme for operation over time-varying channels. The motivation for the work stems from its application to signal compression and transmission over broadcast and mobile communication channels. The basic system consists of a multi-stage vector quantizer whose stages are unequally protected. For the sake of simplicity, the discussion was restricted to the case of a particular unequal protection scheme which is appropriate for the CDMA scenario. We described a novel codebook search and design procedure which exploits the varying level of protection provided to the different stages. The performance results were presented for the case of broadcast and Nakagami fading channels. Compared to a standard equal protection scheme, the proposed scheme was shown to achieve substantial performance gains. Finally we described the weighted decoding technique, where we improve the reconstructed vector taking into account the estimates of the decoded bit error rate for each stage index. The implementation of weighted decoding is feasible and results in considerable performance enhancement. The overall gains over standard equal error protection MSVQ are in the range of 3-5 dB.

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