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On Optimum Communication Cost for Joint Compression and Dispersive Information Routing

Kumar Viswanatha, Emrah Akyol and Kenneth Rose
Department of Electrical and Computer Engineering
University of California at Santa Barbara, CA-93106
Email: {kumar, eakyol, rose}@ece.ucsb.edu

***Abstract*—In this paper, we consider the problem of minimum cost joint compression and routing for networks with multiple-sinks and correlated sources. We introduce a routing paradigm, called dispersive information routing, wherein the intermediate nodes are allowed to forward a subset of the received bits on subsequent paths. This paradigm opens up a rich class of research problems which focus on the interplay between encoding and routing in a network. What makes it particularly interesting is the challenge in encoding sources such that, exactly the required information is routed to each sink, to reconstruct the sources they are interested in. We demonstrate using simple examples that our approach offers better asymptotic performance than conventional routing techniques. We also introduce a variant of the well known random binning technique, called 'power binning', to encode and decode sources that are dispersively transmitted, and which asymptotically achieves the minimum communication cost within this routing paradigm.**

I. Introduction

Signal compression of correlated sources for transmission through multi-hop networks has recently attracted much attention in the research community, primarily due to its direct application in sensor networks. This paper considers the problem of minimum cost communication in a multi-hop network with multiple-sinks and correlated sources. Research related to compression in networks can broadly be classified into two camps. The first approach performs compression at intermediate nodes without resorting to distributed source coding (DSC) techniques. Such techniques tend to be wasteful at all but the last hops of the communication path. The second approach performs DSC followed by simple routing. Well designed DSC followed by optimal routing can provide good performance gains. This paper focuses on the latter category.

Multi-terminal source coding has one of its early roots in the seminal work of Slepian and Wolf [1]. They showed, in the context of lossless coding, that side-information available only at the decoder can nevertheless be fully exploited as if it were available to the encoder, in the sense that there is no asymptotic performance loss. Later, Wyner and Ziv [2] derived a lossy coding extension that bounds the rate-distortion performance in the presence of decoder side information. Extensive work followed considering different network scenarios and obtaining achievable rate regions for them, including [3], [4], [5]. Han and Kobayashi [6] extended the Slepian-Wolf result to general multi-terminal source coding scenarios. For a multi-sink network, with each sink requesting for a subset of sources, they characterized an achievable rate region for lossless reconstruction of all the requested sources at each sink. Csiszar and Korner [7] provided an alternate, but equivalent characterization of the achievable rate region.

There has also been a considerable amount of work on joint compression-routing for networks. A survey of routing techniques in sensor networks is given in [8]. [9] compared different joint compression-routing schemes for a correlated sensor grid and also proposed an approximate, practical, static source clustering scheme to achieve compression efficiency. Cristascu et.al [10] considered joint optimization of Slepian-Wolf coding and a routing mechanism, we call 'broadcasting'[1], wherein each source broadcasts its information to all sinks that intend to reconstruct it. Such a routing mechanism is motivated from the extensive literature on optimal routing for independent sources [11]. [12] proved the general optimality of that approach for networks with a single sink. Recently, [13] demonstrated its sub-optimality for the multi-sink scenario. This paper takes a step further towards finding the best joint compression-routing mechanism for a multi-sink network. We note the existence of a volume of work on network coding for correlated sources, eg. [14], [15]. But the routing mechanism we introduce in this paper does not require possibly complex network coders at intermediate nodes, and can be realized using simple conventional routers. The approach does have potential implications on network coding, but these are beyond the scope of this paper.

The new routing paradigm we introduce, which we call, "dispersive information routing" (DIR), is designed to forward only the required information to each sink. We show from basic principles that DIR achieves a lower communication cost compared to broadcasting in a network, wherein the sinks usually receive more information than they need. In what follows we first motivate the routing paradigm using a simple example. We also give the basic intuition for the encoding scheme that achieves minimum communication cost. We then formulate and solve using a general setting to find the minimum cost achievable by DIR.

[1]Note that we loosely use the term 'broadcasting' instead of 'multicasting' to stress the fact that ***all*** the information transmitted by any source is routed to every sink that reconstructs the source. Also, our approach to routing is in some aspects, a variant of multicasting.

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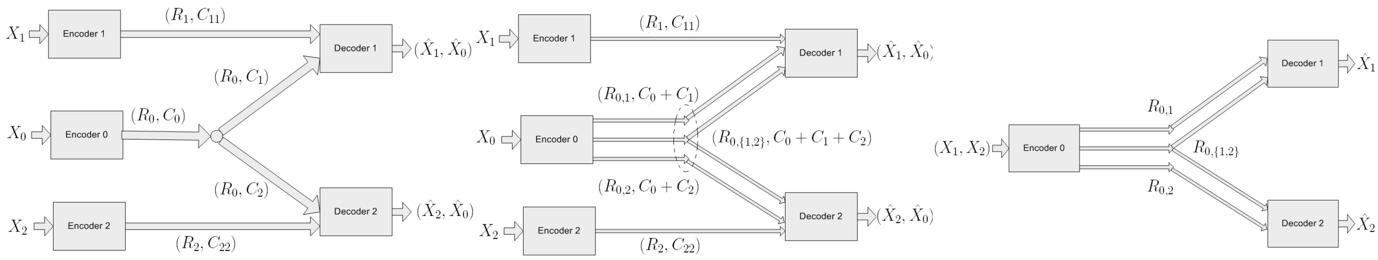


(a) Broadcasting (b) DIR (c) Wyner's Setup

Fig. 1. Figure (a) shows the example considered. Figure (b) shows how dispersive information routing at the collector can be realized using a conventional router - routing 3 smaller packets. Figure (c) depicts the resemblance between the DIR setup and the Wyner's setup.

II. Motivating Example

Consider the network shown in figure 1a. There are three sources X_0, X_1 and X_2 and two sinks S_1 and S_2. Sink S_1 reconstructs the source pair (X_0, X_1), while S_2 reconstructs (X_0, X_2). Source X_0 communicates with the two sinks through an intermediate node (called the 'collector') which is functionally a simple router. The edge weights on each path in the network are shown in the figure. The cost of communication through a link is a function of the bit rate flowing through it and the edge weight, which we will assume for simplicity to be a simple product $f(r, c) = rc$ in this paper, noting that the approach is directly extendible to more complex cost functions. The objective is to find the minimum communication cost for lossless reconstruction of respective sources at the sinks.

We first consider the communication cost when broadcasting is employed [10] wherein the routers forward all the bits received from a source to all the decoders that will reconstruct it. In other words, routers are not allowed to "split a packet" and forward a portion of the received information. Hence the branches connecting the collector to the two decoders carry the same rates as the branch connecting encoder 0 to the collector. We denote the rate at which X_0, X_1 and X_2 are encoded by R_0, R_1 and R_2 respectively.

Using results in [10], it can be shown that the minimum communication cost under broadcasting is given by the following linear programming formulation:

$$C_b = \min\{(C_0 + C_1 + C_2)R_0 + C_{11}R_1 + C_{22}R_2\} \tag{1}$$

under the constraints:

$$\begin{aligned} R_1 \geq H(X_1|X_0)\,, \qquad & R_0 \geq H(X_0|X_1) \\ R_2 \geq H(X_2|X_0)\,, \qquad & R_0 \geq H(X_0|X_2) \\ R_1 + R_0 &\geq H(X_0, X_1) \\ R_2 + R_0 &\geq H(X_0, X_2) \end{aligned} \tag{2}$$

To gain intuition into dispersive information routing, we also consider a special case of the network when the branch weights are such that $C_{11}, C_{22} \ll C_0, C_1, C_2$. Let us specialize the above equations for this case. The constraint $C_{11}, C_{22} \ll C_0, C_1, C_2$, forces sources X_1 and X_2 to be encoded at rates $R_1 = H(X_1)$ and $R_2 = H(X_2)$, respectively. Therefore, this scenario effectively captures the case when sources X_1 and X_2 are available at decoders 1 and 2, respectively, as side information. From equations (1) and (2) for minimum communication cost, X_0 is encoded at a rate:

$$R_0^* = \max\{H(X_0|X_1), H(X_0|X_2)\} \tag{3}$$

and therefore the minimum communication cost is given by:

$$\begin{aligned} C_b^* = (C_0 + C_1 + C_2) \max\{(H(X_0|X_1), H(X_0|X_2))\} \\ + C_{11}H(X_1) + C_{22}H(X_2) \end{aligned} \tag{4}$$

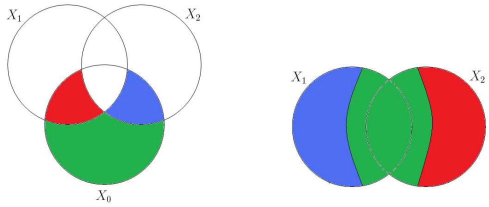
Is this the best we can do? The collector has to transmit enough information to decoder 1 for it to decode X_0 and hence the rate is at least $H(X_0|X_1)$. Similarly on the branch connecting the collector to decoder 2 the rate is at least $H(X_0|X_2)$. But if $H(X_0|X_1) \neq H(X_0|X_2)$, there is excess rate on one of the branches.

Let us now relax this restriction and allow the collector node to "split" the packet and route different subsets of the received bits on the forward paths. We could equivalently think of the encoder 0 transmitting 3 smaller packets to the collector; first packet has a rate $R_{0,\{1,2\}}$ bits and is destined to both sinks. Two other packets have rates $R_{0,1}$ and $R_{0,2}$ and are destined to sinks 1 and 2 respectively. Technically, in this case, the collector is again a simple conventional router.

We call such a routing mechanism, where each intermediate node transmits a subset of the received bits on each of the forward paths "Dispersive Information Routing" (DIR). Note that unlike network coding, DIR does not require expensive coders at intermediate nodes, but rather can always be realized using conventional routers with each source transmitting multiple packets into the network intended to different subsets of sinks. Therefore, hereafter, we interchangeably use the concepts of "packet splitting" at intermediate nodes and conventional routing of smaller packets, noting the equivalence in the achievable rates and costs. This scenario is depicted in figure 1b with the modified costs each packet encounters.

Two obvious questions arise - Does DIR achieve a lower communication cost compared to broadcasting? If so, what is the minimum communication cost under DIR?

We first aim to find the minimum cost using DIR if $C_{11}, C_{22} \ll C_0, C_1, C_2$ (i.e. $R_1 = H(X_1)$ and $R_2 = H(X_2)$). To establish the minimum cost one may



(a) DIR (b) Wyner's setup

Fig. 2. Venn Diagram - Blue indicates what is needed by decoder 1 alone, red indicates what is needed by decoder 2 alone and green in the shared information. Figure (a) shows the diagram for the DIR setting and figure (b) for the Wyner's setting.

first identify the complete achievable rate region for the rate tuple $\{R_{0,1}, R_{0,\{1,2\}}, R_{0,2}\}$ for lossless reconstruction of X_0 at both the decoders. Then one finds the rate point that minimizes the total communication cost, determined using the modified weights shown in figure 1b. Before attempting a final solution, it is worthwhile to consider one operating point, $P_1 = \{R_{0,1}, R_{0,\{1,2\}}, R_{0,2}\} = \{I(X_2; X_0|X_1), H(X_0|X_1, X_2), I(X_1; X_0|X_2)\}$ and provide the coding scheme that achieves it. Extension to other "interesting points" and to the whole achievable region follows in similar lines. This particular rate point is considered first due to its intuitive appeal as shown in a Venn diagram (figure 2).

Wyner considered a closely resembling network [5] shown in figure 1c. In his setup, the encoder observes 2 sources (X_1, X_2) and transmits 3 packets (at rates $R_{0,1}, R_{0,\{1,2\}}, R_{0,2}$ respectively), one meant for each subset of sinks. The two sinks reconstruct sources X_1 and X_2 respectively. He showed that, the rate tuple $\{R_{0,1}, R_{0,\{1,2\}}, R_{0,2}\} = \{H(X_1|X_2), I(X_1; X_2), H(X_2|X_1)\}$ is not achievable in general and that there is a rate loss due to transmitting a common bit stream; in the sense that individual decoders must receive more information than they need to reconstruct their respective sources. Wyner defined the term "Common Information", here denoted by $W(X_1; X_2)$ as the minimum rate $R_{0,\{1,2\}}$ such that $\{R_{0,1}, R_{0,\{1,2\}}, R_{0,2}\}$ is achievable and $R_{0,1} + R_{0,\{1,2\}} + R_{0,2} = H(X_1, X_2)$. He also showed that $W(X_1; X_2) = \inf I(X_1, X_2; W)$ where the inf is taken over all auxiliary random variables W such that $X_1 \rightarrow W \rightarrow X_2$ form a Markov chain. Wyner showed that in general $I(X_1; X_2) \leq W(X_1; X_2) \leq \max(H(X_1), H(X_2))$. We note in passing, an earlier definition of common information [16] which measures the maximum shared information that can be fully utilized by both the decoders. It is less relevant to dispersive information routing.

At a first glance, it might be tempting to extend Wyner's argument to the DIR setting and say P_1 is not achievable in general, i.e., each decoder has to receive more information than it needs. But interestingly enough, a rather simple coding scheme achieves this point and simple extensions of the coding scheme can achieve the entire rate region. Note that in this section, we only provide intuitive arguments to validate the result. We derive a variant of the "random binning paradigm" in section III for the general setup.

We focus on encoder 0, assuming that encoders 1 and 2 transmit at the respective source entropies. Encoder 0 observes a sequence of n realizations of the random variable X_0. This sequence belongs to the typical set, τ_ϵ^n, with high probability. Every typical sequence is assigned 3 indices, each independent of the other. The three indices are assigned using uniform pmfs over $[1:2^{nR_{0,1}}]$, $[1:2^{nR_{0,\{1,2\}}}]$ and $[1:2^{nR_{0,2}}]$ respectively. All the sequences with the same first index, m_1, form a bin $\mathcal{B}_1(m_1)$. Similarly bins $\mathcal{B}_2(m_2)$ and $\mathcal{B}_3(m_3)$ are formed for indices m_2 and m_3. Upon observing a sequence $X_0^n \in \tau_\epsilon^n$ with indices m_1, m_2 and m_3, the encoder transmits index m_1 to decoder 1 alone, index m_3 to decoder 2 alone and index m_2 to both the decoders.

The first decoder receives indices m_1 and m_2. It tries to find a typical sequence $\hat{X}_0^n \in \mathcal{B}_1(m_1) \cap \mathcal{B}_2(m_2)$ which is jointly typical with the decoded information sequence X_1^n. As the indices are assigned independent of each other, every typical sequence has uniform pmf of being assigned to the index pair $\{m_1, m_2\}$ over $[1:2^{n(R_{0,1}+R_{0,\{1,2\}})}]$. Therefore, having received indices m_1 and m_2, using counting arguments similar to Slepian and Wolf [1], [4], the probability of decoding error asymptotically approaches zero if:

$$R_{0,1} + R_{0,\{1,2\}} \geq H(X_0|X_1) \tag{5}$$

Similarly, the probability of decoding error approaches zero at the second decoder if:

$$R_{0,2} + R_{0,\{1,2\}} \geq H(X_0|X_2) \tag{6}$$

Clearly (5) and (6) imply that P_1 is achievable. In similar lines to [1], [4], the above achievable region can also be shown to satisfy the converse and hence is the complete achievable rate region for this problem. We refer to such a binning approach as 'Power Binning' as multiple independent indices are assigned to each (non-trivial) subset of the decoders - power set. Also note that the difference in Wyner's setting was that the two sources were to be encoded jointly for separate decoding of each source. But in our setup, source X_0 is to be encoded for lossless decoding at both the decoders.

The minimum cost operating point is the point that satisfies equations (5) and (6) and minimizes the cost function:

$$C^*_{\text{DIR}} = \min \big\{(C_0 + C_1)R_{0,1} + (C_0 + C_2)R_{0,2} + (C_0 + C_1 + C_2)R_{0,\{1,2\}}\big\} \tag{7}$$

The solution is either one of the two points $P_2 = \{0, H(X_0|X_1), H(X_0|X_2) - H(X_0|X_1)\}$ or $P_3 = \{H(X_0|X_1) - H(X_0|X_2), H(X_0|X_2), 0\}$ and both achieve lower total communication cost compared to broadcasting (C_b^* - equation (4)) for any $C_0, C_1, C_2 \gg C_{11}, C_{22}$. Not surprisingly, the operating point is within the Han and Kobayashi achievable rate region [6] (where network costs and routing constraints are ignored).

The above coding scheme can be easily extended to the case of arbitrary edge weights. The rate region for the tuple

$\{R_1, R_2, R_{0,1}, R_{0,\{1,2\}}, R_{0,2}\}$ and the cost function to be minimized are given by:

$$C_{\mathrm{DIR}} = \min\{C_{11}R_1 + C_{22}R_2 + (C_0 + C_1)R_{0,1} + (C_0 + C_2)R_{0,2} + (C_0 + C_1 + C_2)R_{0,\{1,2\}}\} \tag{8}$$

under the constraints:

$$\begin{aligned} R_1 &\geq H(X_1|X_0) \\ R_{0,1} + R_{0,\{1,2\}} &\geq H(X_0|X_1) \\ R_1 + R_{0,1} + R_{0,\{1,2\}} &\geq H(X_0, X_1) \\ R_2 &\geq H(X_2|X_0) \\ R_{0,2} + R_{0,\{1,2\}} &\geq H(X_0|X_2) \\ R_2 + R_{0,2} + R_{0,\{1,2\}} &\geq H(X_0, X_2) \end{aligned} \tag{9}$$

If $R_1 = H(X_1)$ and $R_2 = H(X_2)$ (9) specializes to (5) and (6). Also, it can be easily shown that the total communication cost obtained as a solution to the above formulation is always lower than that for broadcasting, C_b (equations (1) and (2)) if $C_0, C_1, C_2 > 0$.

III. General Problem setup and Solution

A. Problem Formulation

Let a network be represented by an undirected graph $G = (V, E)$. Each edge $e \in E$ is a network link whose communication cost depends on the edge weight w_e. The nodes V consist of N source nodes, M sinks, and $|V|-N-M$ intermediate nodes. Source node i has access to source random variable X_i distributed over alphabet $\mathcal{X}_i$. The joint probability distribution of $(X_1 \ldots X_N)$ is known at all the nodes. The sinks are denoted $S_1, S_2 \ldots, S_M$. A subset of sources are to be reconstructed (losslessly) at each sink. Let the subset of source nodes to be reconstructed at sink S_j be $V^j \subseteq V$. Conversely, source i has to be reconstructed at a subset of sinks denoted by $S^i \subseteq \{S_1, S_2 \ldots, S_M\}$ [2]. We denote the set $\{1 \ldots N\}$ by Σ and the set $\{1 \ldots M\}$ by Π. The objective is to find the minimum communication cost achievable using dispersive information routing at all intermediate nodes in the network. Note that, in this paper, we assume that only sources to be reconstructed at any sink communicate with the sink (i.e., there are no 'helpers' [7]). The more general case of DIR with every source (possibly) communicating with every sink will be addressed in the sequel. The general setting in the context of conventional routing was addressed in [13].

Hereafter, we use the following notation. For any random variable X, we use X^n to represent n independent realizations of the random variable and the corresponding alphabet by $\mathcal{X}^n$. For any set s, $|s|$ denotes the cardinality of the set and 2^s denotes the power set. $2^s \backslash \phi$ denotes all the non-empty subsets of the set s. For any set $s = \{k_1, k_2 \ldots k_{|s|}\} \subseteq \Sigma$ we use X_s to denote $\{X_i : i \in s\}$ and the corresponding alphabet $\mathcal{X}_{k_1} \times \mathcal{X}_{k_2} \times \ldots \mathcal{X}_{k_{|s|}}$ by $\mathcal{X}_s$.

[2]Note that the case of side information at the decoder can be trivially included in this formulation with $w_e = 0$ on the branch connecting the side information source and the decoder.

B. Obtaining modified costs

DIR requires each source i to transmit a packet to every set of sinks that reconstruct X_i, i.e., one packet to all $s \in 2^{S^i} \backslash \phi$. Denote the packets transmitted by encoder i by $P^i_1, P^i_2 \ldots P^i_{|2^{S^i} \backslash \phi|}$. Let E^i_s be the set of all paths from source i to the subset of sinks $s \in 2^{S^i} \backslash \phi$. The optimum route for packet P^i_s from the source to these sinks is determined by a spanning tree optimization (minimum Steiner tree) [11]. More specifically, for each packet P^i_s, the optimum route is obtained by minimizing the cost over all trees rooted at node i which span all sinks $S_j \in s$. The minimum cost of transmitting packet P^i_s with $R_{i,s}$ bits from source i to the subset of sinks s, denoted by $d_i(s)$, is given by:

$$d_i(s) = R_{i,s} \min_{Q \in E^i_s} \sum_{e \in Q} w_e \tag{10}$$

Having obtained the modified costs for each packet in the network, our next aim is to find the rate region and the minimum communication cost will then follow directly from a simple linear programming formulation.

C. Entire rate region

An $\epsilon-$DIR code $(f_1, f_2 \ldots f_N, h_1, h_2 \ldots h_M)$ of block length n for the sources $X_1, X_2 \ldots X_N$ for given $V^j \ \forall j \in \Pi$, is the following set of mappings:

- The encoders : $f_i : \mathcal{X}^n_i \rightarrow \{(0,1)^{M^i_s}\} \ \forall i \in \Sigma, \ s \in 2^{S^i} \backslash \phi$, where M^i_s are positive integers. Packet P^i_s has M^i_s bits in it and is routed from source i to the subset of sinks s.
- The decoders : $h_j : (0,1)^{M_j} \rightarrow \mathcal{X}^n_{V^j} \ \forall j \in \Pi,$ where $(0,1)^{M_j}$ is the set of all possible bit sequences received by decoder S_j. Denote by $M_{i,j}$, the total number of bits transmitted from source i to sink S_j. i.e.:

$$M_{i,j} = \sum_{s \in 2^{S^i} \backslash \phi, \ s \ni j} M^i_s \tag{11}$$

Then M_j is the total number of bits received by decoder S_j and is given by:

$$M_j = \sum_{i \in V^j} M_{i,j} \tag{12}$$

A rate tuple $\{R^i_s\} \ \forall i \in \Sigma, \ s \in 2^{S^i} \backslash \phi$ is said to be achievable, if there exists an $\epsilon-$DIR code with all the mappings defined as above and satisfying:

$$\Pr\left[X^n_{V^j} \neq h_j\left(\cup_{i \in V^j} f_i(X^n_i)\right)\right] < \epsilon \tag{13}$$

$$M^i_s < n(R^i_s + \epsilon) \tag{14}$$

Define R^*_{DIR} to be the set of rate tuples that satisfy the following constraints $\forall j \in \Pi$ and $\forall t \in 2^{V^j} \backslash \phi$:

$$\sum_{i \in t} \sum_{s \in 2^{S^i} \backslash \phi, \ s \ni j} R^i_s \geq H\left(X_t | X_{V^j \backslash t}\right) \tag{15}$$

Theorem. *R^*_{DIR} is the entire rate region.*

Proof: Codebook design and power binning: At encoder i, associate each typical sequence $X_i^n \in \tau_\epsilon^n$ with $2^{S^i}\backslash\phi$ independently generated indices, each according to a uniform pmf over $[1\dots 2^{nM_s^i}]$. The indices are denoted by m_s^i $\forall s \in 2^{S^i}\backslash\phi$. All sequences which are assigned the same k'th index m are said to fall in the same bin $\mathcal{B}_k^i(m)$ $\forall k \in \{1\dots 2^{S^i}\backslash\phi\}$ and $\forall m \in \{1\dots 2^{nM_s^i}\}$.

Encoding: Each encoder observes n realizations of the random variable X_i. If $X_i^n \in \tau_\epsilon^n$, it transmits index $m_s^i \in \{1\dots 2^{nM_s^i}\}$ to the subset of sinks s. Therefore the rate of packet from source i to the subset of sinks s is M_s^i. Remember that this packet encounters a total cost of $d_i(s)$ before reaching the sinks. If $X_i^n \notin \tau_\epsilon^n$ the encoder transmits index 1 to all $s \in 2^{S^i}\backslash\phi$.

Decoding: Each decoder j receives all indices m_s^i such that $s \ni j$ and $i \in V^j$. The decoder tries to find a jointly typical sequence tuple $\{\hat{X}_i : i \in V^j\}$ such that $\hat{X}_i \in \cap_{s\in 2^{S^i}\backslash\phi,\ s\ni j}\mathcal{B}_s^i(m_s^i)$. If it does not find any jointly typical sequence tuple, it declares an error.

Error Analysis: An error occurs due to one of the causes:
(1) Any encoder observes $X_i^n \notin \tau_\epsilon^n$. The probability of this event is $< \epsilon$ for sufficiently large n by the weak law of large numbers.
(2) Any decoder fails to find a jointly typical sequence tuple: We denote the index tuple, $\{m_s^i : s \ni j\}$ by $m_{i,j}$. As all the indices are independent of each other and are drawn from uniform pmf's, each typical sequence X_i^n is assigned $m_{i,j}$ with a uniform pmf over $[1\dots 2^{nM_{i,j}}]$. Decoder j receives $m_{i,j}$ $\forall i \in V^j$. From arguments similar to [4], [1], the probability of decoder error at decoder j is $< \epsilon$ if for all $t \in 2^{V^j}\backslash\phi$:

$$\sum_{i\in t} M_{i,j} \geq n(H\left(X_t|X_{V^j\backslash t}\right) + \epsilon) \tag{16}$$

The achievable rate region given in (15) follows directly by substituting (11) in (16). Also note that at each decoder, the converse follows similarly to the converse to the usual Slepian and Wolf setup. Hence, R^*_{DIR} is the entire rate region. ■

It is worthwhile to note that the same rate region can be obtained by applying results of Han and Kobayashi [6], assuming $|2^{S^i}\backslash\phi|$ independent encoders at each source, albeit with a more complicated coding scheme involving multiple auxiliary random variables. But, Han and Kobayashi ignore the network routing and cost constraints in their formulation and hence have no motivation for the encoders to transmit multiple packets into the network.

D. Finding the Minimum Cost

The minimum cost follows directly from a simple linear programming formulation:

$$\min_{\bar{R}\in R^*_{DIR}} \sum_{i=1}^{N}\sum_{s=1}^{|2^{S^i}\backslash\phi|} R_s^i \times d_i(s) \tag{17}$$

It can be easily seen that the minimum cost achievable using DIR is lower than broadcasting for most source distributions.

IV. Conclusion and Future work

In this paper we addressed the problem of optimizing the communication cost for a general network with multiple sinks and correlated sources under a routing paradigm called dispersive information routing. Unlike network coding, such a routing mechanism can always be realized using conventional routers with sources transmitting multiple packets, each meant for a subset of sinks. We proposed a coding scheme that asymptotically achieves the optimum cost under the routing paradigm. Future work includes extending the work to the more general case where sources may communicate with sinks that do not reconstruct them, and designing practical (finite delay) joint coder-routers that achieve low communication costs.

Acknowledgments

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