On Random Binning versus Conditional Codebook Methods in Multiple Descriptions Coding

Emrah Akyol, Kumar Viswanatha, and Kenneth Rose
{eakyol, kumar, rose}@ece.ucsb.edu
University of California, Santa Barbara

***Abstract*—There are two common types of encoding paradigms in multiple descriptions (MD) coding: i) an approach based on conditional codebook generation, which was originally initiated by El-Gamal and Cover for the 2 channel setting and later extended to more than 2 channels by Venkataramani, Kramer and Goyal (VKG), ii) and an approach based on Slepian and Wolf's random binning technique, proposed by Pradhan, Puri and Ramchandran (PPR) for $L > 2$ descriptions. It is well known that the achievable region due to PPR subsumes the VKG region for the symmetric Gaussian MD problem. Motivated by several practical advantages of random binning based methods over the conditional codebook encoding, this paper focuses on the following important questions: Does a random binning based scheme achieve the performance of conditional codebook encoding, even for the 2 descriptions scenario? Are random binning based approaches beneficial for settings that are not fully symmetric? This paper answers both these questions in the affirmative. Specifically, we propose a 2 descriptions coding scheme, based on random binning, which subsumes the currently known largest region for this problem due to Zhang and Berger. Moreover, we propose its extensions to $L > 2$ channels and derive the associated achievable regions. The proposed scheme enjoys the advantages of both encoding paradigms making it particularly useful when there is symmetry only within a subset of the descriptions.**

***Index Terms*—Multiple description coding, source coding, rate-distortion theory**

I. Introduction

The multiple descriptions (MD) coding problem is a long standing open problem in source coding [1], [2], [3]. The objective is to encode the source into multiple descriptions, such that when a subset of the descriptions is received at the decoder, the source is estimated within a prescribed distortion. For the two descriptions setting, the rate-distortion region has been completely characterized for the Gaussian source under mean squared error (MSE) distortion metric [1], and the encoding scheme due to El Gamal and Cover (EGC) [2] is known to be optimal. This success of the EGC scheme led to the natural conjecture that EGC scheme is optimal for all sources and distortion measures. However, Zhang and Berger (ZB) [3] proposed an extension to the EGC scheme in which a common codeword is sent in both the descriptions, and showed that this scheme achieves points strictly outside the EGC region for a binary source under Hamming distortion measure. The complete characterization of the two descriptions problem for general sources and distortion measures remains an open problem.

Recent focus has shifted to L-description problem ($L > 2$). Venkataramani, Kramer and Goyal (VKG) [4] extended the conditional codebook approach of [3] to L descriptions using one common codeword that is broadcasted to all the descriptions along with a combinatorial number refinement codewords, that are split across the descriptions and are used to refine the reconstructions when multiple descriptions are received. Recently, we further extended this coding scheme by realizing the need for a combinatorial number of common codewords, and proposed a new achievable region based on the "combinatorial message sharing" (CMS) principle [5]. The basic advantage of CMS over VKG scheme is its flexible adaptation to non-symmetric rates and distortions, which allows it to achieve points which are strictly outside the VKG region.

An interesting alternative encoding paradigm, based on Slepian-Wolf's random binning techniques, was proposed by Puri, Pradhan and Ramchandran (PPR) [6], [7] for an important special case of the $L-$descriptions problem - the symmetric MD, where all the descriptions are encoded at the same rate and the distortion depends only on the number of descriptions received, and not on the specific subset. They derived an achievable rate-distortion region under these assumptions and showed that random binning helps in exploiting the symmetry, which the conditional codebook based schemes fail to utilize. The high level intuition in support of binning is that the encoder is not aware of which descriptions are received at the decoder, but this information is available at the decoder and can be utilized as side information, following the seminal ideas of Slepian and Wolf. In [6], a coding scheme based on source-channel erasure codes (SCEC) was proposed, which have similar structure to the (L, k) maximum distance separable codes (MDS) used in channel coding. In [7], SCEC codes were sequentially layered from (L, L) to $(L, 1)$ resulting in a scheme that we call the multilayer random binning scheme. A variant of this coding scheme achieves several cross sections of the outer bound for a Gaussian source under MSE in this fully symmetric setting, as shown in a series of publications by Wang and Viswanath [8], [9] and approximates the outer bound in general [10].

In this paper, we consider utilizing binning ideas in settings that are not fully symmetric. We begin with a two descriptions problem. A fundamental theoretical question pertinent to this setting is: Is random binning based coding compatible with the

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best known achievable coding scheme? As we show in Section III, a rather simple scheme based on random binning leads to an achievable region that subsumes the ZB region. Beyond its theoretical importance, this result has some important practical implications. Conditional codebook based encoding of the type used in ZB, VKG and CMS, requires an exponentially large number of codebooks as an independent codebook is generated conditioned on every codeword tuple from the previous layers. This fact makes conditional codebook encoding practically infeasible, or at least undesirable. Moreover, for Gaussian sources, random binning can be easily realized with structured schemes such as nested lattice codes [11]. However, no such structured scheme is known for conditional codebook encoding.

Having established this result, we next incorporate random binning within our recent CMS framework to extend the proposed scheme to $L > 2$ descriptions. Specifically, we perform random binning within codebooks structured according to the CMS principle for the common codeword layers, while setting all enhancement layers to a constant, i.e., we do not use any enhancement layer. We first present an achievable rate-distortion region of this simple random binning based method.

Finally, we utilize layered random binning within CMS structured codebooks, i.e., we incorporate a PPR-like multi-layer random binning scheme for each subset of the combinatorially organized common layer codebooks, which efficiently utilize the symmetry using random binning, while maintaining adaptability to partial asymmetry in rates and distortions. This new scheme can be viewed a nontrivial generalization of the PPR schemes and hence includes their region as an extreme special case. Specifically, if we set the CMS common layer codebooks to a constant, then we obtain the multilayer PPR region. Also, it is intuitively expected that this new scheme will outperform the conditional codebook based schemes due to the inherent advantage of PPR approach over conditional codebook based techniques in efficiently utilizing the underlying symmetry.

As an example setting to provide intuition for the potential benefits of the proposed scheme, consider a scenario where symmetry is limited to a subset of the description rates, say, $R_1 > R_2 = R_3 = R_4 = R$. Then, a PPR-like scheme would bin the codebooks at the same rate of R, due to its highly symmetric structure. However, an excess rate of $R_1 - R$ bits would remain unused. On the other extreme, a conditional codebook based scheme would encode all descriptions at their respective full rates, but would fail to utilize the underlying partial symmetry of descriptions $2, 3$ and 4. The proposed paradigm, which uses multilayer random binning for each subset, efficiently utilizes the available partial symmetry while at the same time adapts itself to the asymmetry in overall rates leading to improved performance.

This paper is organized as follows: In Section II, we present the notation and an overview of prior MD schemes. In Section III, we describe the new approach along with the new achievable region for this scheme. Section IV provides conclusions.

II. Preliminaries

A. Notation

Let $\{X(i)\}_{i=1,2,...}$ be a zero-mean memoryless and stationary source taking values in alphabet $\mathcal{X}$ and let $\hat{\mathcal{X}}$ be the reconstruction alphabet. The vector $[X(1), X(2), ..., X(n)]$ is compactly denoted by x^n. The distortion measure $d : \mathcal{X} \times \hat{\mathcal{X}} \rightarrow \mathbb{R}^+$ is bounded and additive. We use the notation $\mathcal{I}_L$ to denote the set $\{1, 2, .., L\}$. For parsimony of notation, we employ $H(X)$ to denote the entropy of a discrete random variable X, or differential entropy if X is continuous. For an arbitrary set $\mathcal{A}$, we use $2^{\mathcal{A}}_m$ to denote the set of all subsets of $\mathcal{A}$ with cardinality greater or equal to m, i.e.,

$$2^{\mathcal{A}}_m \triangleq \{\mathcal{S} : \mathcal{S} \subseteq \mathcal{A}, |\mathcal{S}| \geq m\}.$$

B. Two descriptions

The first two descriptions coding scheme was proposed in the seminal paper by El Gamal and Cover [2]. For a fixed joint density $P(X, U_1, U_2)$, their coding scheme generates two random codebooks according to p_{U_1} and p_{U_2} respectively. On observing a typical sequence of X, the encoder finds one codeword from each codebook that are jointly typical with the source sequence. The **EGC Region** region is the convex closure of all rate-distortion tuples satisfying:

$$\begin{aligned} D_{\mathcal{A}} &\geq \mathbb{E}\{d(X, g_{\mathcal{A}}(U_{2^{\mathcal{A}}_1}))\}, \quad \mathcal{A} = \{1, 2, 12\} \\ R_i &\geq I(X; U_i), \quad i = 1, 2 \\ R_1 + R_2 &\geq I(X; U_1, U_2) + I(U_1, U_2) \end{aligned}$$

over the joint density of X, U_1, U_2. Note that the original encoding scheme by El Gamal and Cover also includes a refinement layer codeword, decoded only when both of the descriptions are received. It was recently discovered in [12] that this refinement layer is unnecessary in the two descriptions setting and hence can be discarded.

Zhang and Berger proposed a modified MD coding scheme [3] that differs from the EGC scheme in the addition of a common codeword, which is sent in both of the descriptions. Clearly, the ZB region subsumes EGC by construction. However, Zhang and Berger showed that for binary source under the Hamming distortion measure, it achieves points strictly outside the EGC region. We briefly summarize the ZB coding scheme here. It first generates a codebook for U_{12} with the marginal $p_{U_{12}}$. For each codeword of U_{12}, it generates a codebook for U_1 and U_2 according to their respective conditional densities $p_{U_1|U_{12}=u_{12}}$ and $p_{U_2|U_{12}=u_{12}}$. On observing a typical sequence of X, the encoder tries to find a jointly typical codeword tuple one from each codebook. The codeword index of U_{12} is sent in both descriptions, while indices for U_1 and U_2 are sent in descriptions 1 and 2 respectively. The **ZB Region**, denoted by $\mathcal{R}_{ZB}$, is the convex closure of all rate-distortion tuples satisfying

$$\begin{aligned} D_{\mathcal{A}} \geq& \mathbb{E}\{d(X, g_{\mathcal{A}}(U_{2^{\mathcal{A}}_1}))\}, \quad \mathcal{A} = \{1, 2, 12\} \\ R_i \geq& I(X; U_{12}, U_i) \quad i = 1, 2 \\ R_1 + R_2 \geq& 2I(X; U_{12}) + I(X; U_1, U_2|U_{12}) + I(U_1; U_2|U_{12}) \end{aligned}$$

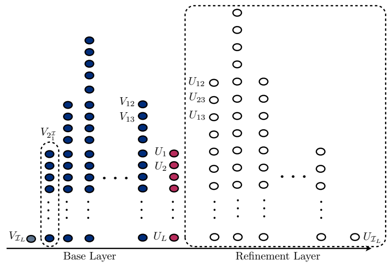

Fig. 1. Codebook structure in the CMS scheme

over the joint density of X, U_1, U_2, U_{12}.

C. The Venkataramani-Kramer-Goyal Scheme

The VKG scheme is a generalization to $L \geq 2$ setting, of the conditional codebook schemes originally designed for $L = 2$. The codebook generation follows the order: common layer → base layer → refinement layer. First, the codebook associated with $V_{\mathcal{I}_L}$ is generated with the marginal $p_{V_{\mathcal{I}_L}}$. Conditioned on each codeword from $V_{\mathcal{I}_L}$, the codebooks for U_l where $l \in \mathcal{I}_L$, is generated according to their respective conditional densities. Next, for each $U_{\mathcal{A}}, \mathcal{A} \in 2_2^{\mathcal{A}}$ a single codeword is generated, conditioned on an appropriately chosen subset of codeword tuples from the previous layers. The codeword index of U_l is sent in the description $l \in \mathcal{I}_L$. Along with the private messages, each description also carries a shared message, which is the codeword index of $V_{\mathcal{I}_L}$. Note that $V_{\mathcal{I}_L}$ is the only common layer random variable. U_l where $l \in \mathcal{I}_L$ form the base layer random variables and all $U_{\mathcal{A}}, \mathcal{A} \in 2_2^{\mathcal{I}}$ form the refinement layers.

D. Combinatorial Message Sharing

In [5], we generalized the VKG scheme by including a common codebook for each subset of the descriptions. The order of codebook generation of the auxiliary random variables is shown in Figure 1 which also specifies the conditioning of the combinatorially shared codebooks. This scheme clearly subsumes VKG as a special case and in fact strictly subsumes its achievable region as was shown in [5].

E. Random Binning-Source Channel Erasure Codes (SCEC)

Here, we focus on the symmetric MD coding problem, where all descriptions are encoded at the same rate and distortion is determined by 'how many' rather than, 'which' descriptions are received. Thus, the achievable rate distortion region is an $L+1$ dimensional space of vectors $(R, D_1, D_2, ..., D_L)$ where D_k is the distortion incurred if k descriptions are received, and R is the description rate.

Suppose for now, we are given that at least k descriptions are received. Then one may employ the best rate kR source code and encode it for the L channels using a (L, k) maximum distance separable (MDS) code. Thus, if any k descriptions are received the optimal distortion for this rate is achieved. However, if more than k descriptions are received the decoder does not gain any new information. Such an encoding scheme is equivalent to generating one codebook at rate kR and having it randomly binned for each of the descriptions independently.

Alternatively, one could generate L independent codebooks in order to gain new information for each received description, and randomly bin the codewords so that the decoder can find only one sequence jointly typical with the source. However, it is not obvious at first whether we can still achieve the optimal distortion for a rate kR code. The key result of [6] shows that with the reception of k descriptions one may recover the source coded at rate kR and monotonically decrease the distortion with reception of additional descriptions. We refer to such codes as SCEC. The achievable region $\mathcal{RD}_{PPR1}$ is formalized below.

PPR1 (SCEC) Region: Let $Y_1, ..., Y_L$ be a collection of random variables that are jointly and symmetrically distributed with an arbitrary source random variable X. Then, if at least k descriptions are received and

$$D_{|\mathcal{A}|} \geq \mathbb{E}\{d(X, g_{\mathcal{A}}(Y_{\mathcal{A}})\}$$
$$R \geq \frac{1}{k} H(Y_1, Y_2, ..., Y_k) - \frac{1}{L} H(Y_1, Y_2, ..., Y_L | X)$$

the rate distortion vector $(R, D_k, D_{k+1}, ..., D_L)$ is achievable.

Random Code Generation: Construct a random codebook with p_{Y_i} independently for each Y_i, $i \in \mathcal{I}_L$ by selecting $2^{nR'}$ codewords uniformly.

Random Binning: To each codebook associate 2^{nR} bins, each containing approximately $2^{n(R'-R)}$ codewords.

Encoding: Given a sourceword x^n, find the indices $i_1, i_2, ..., i_L$ such that the corresponding codewords and source $x^n, y_1(i_1), y_2(i_2), ..., y_L(i_L)$ are jointly typical. Transmit the bin index that contains $y_1(i_l)$ over the channel l.

Decoding: For a set of received bin indices indexed by $\mathcal{K}$ the decoder finds for $k_i \in \mathcal{K}$ the $(x^n, y_{k1}, y_{k2}, ..., y_{kL})$ that are jointly typical and each are contained in the bin corresponding to the received index.

F. Layered Random Binning

In [7], a multilayer random binning scheme is proposed, which can be roughly described as the concatenation of (L, k) SCEC's for $k = 1, 2, ..L$. The encoding and decoding are performed sequentially, one layer after the other. The decoding begins with the first layer ($L, 1$ SCEC) which can be done with the reception of any one of the description. Then, if two descriptions are received, the second layer is decoded with the help of the received bin indices and the decoded codewords from the first layer, and so on. It is shown that this scheme is strictly better than the SCEC scheme. We denote the achievable region of this scheme by $\mathcal{R}_{PPR2}$.

III. Proposed Schemes

A. Proposed Two Descriptions Coding Scheme

In this section, we show that a rather simple coding scheme based on random binning can achieve all point in the ZB region. The proposed two description scheme is as follows. First we randomly generate three codebooks with marginal distributions $p_{U_{12}}, p_{U_1}$ and p_{U_2} with $2^{nr_{12}}, 2^{nr_1}, 2^{nr_2}$ codewords respectively. Then, we independently bin each codebook twice with binning rates $r'_{i,j}$, $i \in \{1, 2, 12\}$, $j \in 1, 2$, once for each description. The bin index corresponding to $r'_{i,j}$ is sent in description j. The encoding is done as follows: Given a source-word x^n, we find codewords, one from each codebook u_{12}^n, u_1^n, u_2^n that are jointly typical with x^n. To guarantee that we will find such codewords, we have the following condition on rates from mutual covering lemma [13]:

$$H(U_i|X) \geq H(U_i) - r_i, i \in \mathcal{A} \tag{1}$$

$$H(U_i, U_j|X) \geq H(U_i) - r_i + H(U_j) - r_j, i, j \in \mathcal{A} \tag{2}$$

$$H(U_{12}, U_1, U_2|X) \geq \sum_{i=12,1,2} H(U_i) - r_i \tag{3}$$

where $\mathcal{A} = \{12, 1, 2\}$. For each description, we put together the bin indices for the corresponding codewords. The codeword of u_{12}^n is decoded by all the decoders, while the codewords of u_1^n and u_2^n are decoded only when the individual descriptions are received. Using random binning arguments, we can decode the codewords successfully if

$$H(U_{12}, U_1) \leq H(U_{12}) - (r_{12} - r'_{12,1}) + H(U_1) - (r_1 - r'_{1,1}) \tag{4}$$

$$H(U_{12}, U_2) \leq H(U_{12}) - (r_{12} - r'_{12,2}) + H(U_2) - (r_2 - r'_{2,2}) \tag{5}$$

and description rates are

$$R_j = r'_{12,j} + r'_{1,j} + r'_{2,j}, j = 1, 2 \tag{6}$$

and due to typicality, distortion $D_{\mathcal{A}}$ is achievable for decoder $\mathcal{A}$ if

$$D_{\mathcal{A}} \geq \mathbb{E}\{d_{\mathcal{A}}(X, g_{\mathcal{A}}(U_{2^{\mathcal{A}}}))\}, \quad \forall \mathcal{A} \in 2^{\mathcal{I}_2} \tag{7}$$

The region obtained by this basic random binning scheme $\mathcal{R}_{RB}$ is the convex hull of these rates over $p_{U_{12}}, p_{U_1}, p_{U_2}$, the binning rates r' and codebook rates r.

Theorem 1. *$\mathcal{R}_{ZB} \subseteq \mathcal{R}_{RB}$.*

Proof: Let us set, $r'_{1,2} = r'_{2,1} = 0$ and $r_{12} = I(X; U_{12})$ which might induce a loss of generality. Then, the set of inequalities for encoding become:

$$H(U_i|X, U_{12}) \geq H(U_i) - r_i, i = 1, 2 \tag{8}$$

and

$$H(U_1, U_2|X, U_{12}) \geq H(U_1) - r_1 + H(U_2) - r_2 \tag{9}$$

noting that (8) and (9) render the remaining encoding inequalities redundant since conditioning reduces entropy and if (8) and (9) are satisfied, so are the remaining encoding inequalities. The decoding inequalities become:

$$H(U_{12}, U_1) \leq H(U_{12}|X) + H(U_1) - (r_1 - R_1) \tag{10}$$

$$H(U_{12}, U_2) \leq H(U_{12}|X) + H(U_2) - (r_2 - R_2) \tag{11}$$

Plugging (10) and (11) in (8) and (9), we have

$$R_1 \geq I(X; U_{12}, U_1) \tag{12}$$

$$R_2 \geq I(X; U_{12}, U_2) \tag{13}$$

$$R_1 + R_2 \geq 2I(X; U_{12}) + I(X; U_1, U_2|U_{12}) + I(U_1; U_2|U_{12}) \tag{14}$$

which is $\mathcal{R}_{ZB}$. ■

Remark: The above does not guarantee strict improvement. This question of whether the proposed random binning scheme achieves points outside $\mathcal{R}_{ZB}$ is under investigation.

B. Extension to $L > 2$ Descriptions

The approach can be extended using the CMS codebook structure. We create codebooks with the marginals $p_{U_{\mathcal{A}}}$ for each common layer codebook of the CMS scheme, and randomly bin each codebook with binning rates $r'_{\mathcal{A},j}$ where j is the description index. In each description, only the corresponding bin indices are sent and using the same principles explained for the two-descriptions setting, we derive the rate-distortion regions subject to encoding and decoding constraints. Note that, we do not use refinement layers in this scheme for simplicity. The following theorem presents the achievable region.

Theorem 2. *An achievable rate-distortion region with the proposed MDC scheme is the convex closure of all rate-distortion tuples satisfying*

$$D_{\mathcal{A}} \geq \mathbb{E}\{d(X, g_{\mathcal{A}}(U_{\mathcal{A}}))\} \quad \forall \mathcal{A} \in 2_1^{\mathcal{I}_L} \tag{15}$$

$$R_l = \sum_{k \in 2^{\mathcal{A}}} r'_{k,l} \quad \forall l \in \mathcal{I}_L \tag{16}$$

for some joint distribution, deterministic decoding functions and rates satisfying the following

$$H(U_{\mathcal{A}}|X) \geq \sum_{k \in 2^{\mathcal{A}}} H(U_k) - r_k \tag{17}$$

and

$$H(U_{2_1^{\mathcal{A}}}) \leq \sum_{k \in 2_1^{\mathcal{A}}} \left(H(U_k) - r_k + \sum_{l \in I_{\mathcal{L}}} r'_{k,l} \right) \tag{18}$$

for any $\mathcal{A} \in 2_1^{\mathcal{I}_L}$.

C. Layered Random Binning within CMS

Here, we integrate the layered random binning approach of PPR-2 [7] within the CMS coding structure. Basically, we use the PPR approach of layering (L, k) MDS codes for each subset of the base descriptions and still can use the PPR enhancement descriptions, i.e., layered SCEC. Note that at the base layer, we do not have the luxury of using independent codebooks (hence, we have MDS codes instead of SCEC at the base layer), but we can bin the base layer codewords, independently and at different rates for each description.

For each subset, $\mathcal{A}$ of the channels that a base description broadcasts to, we layer $(|\mathcal{A}|, k)$ MDS codes for $k = 1, 2, .., |\mathcal{A}|$ over the channels in $\mathcal{A}$. For the refinement descriptions, we layer (L, k) SCEC's for $k = 1, 2, ..., L-1$. Decoder $\mathcal{A}$ will decode the refinement codewords generated from the codebooks with $k \leq |\mathcal{A}|$ and subset $\mathcal{K}$ base descriptions of all codebooks with $k \leq |\mathcal{A} \cap \mathcal{K}|$. It then reconstructs the source jointly using all received descriptions.

The following theorem presents the achievable region for the proposed MDC scheme, see [14] for a detailed proof and examples. We call this region, $\mathcal{RD}_{CMS-RB}$.

Theorem 3. *An achievable rate-distortion region with the proposed MDC scheme is the convex closure of all rate-distortion tuples satisfying*

$$D_{\mathcal{A}} \geq \mathbb{E}\{X, g_{\mathcal{A}}(U_{\mathcal{I}_{\Xi},\mathcal{A}}, V_{\mathcal{K}\in 2_1^{\mathcal{A}},\mathcal{I}_{|\mathcal{A}\cap\mathcal{K}|}})\}, \quad \forall \mathcal{A} \in 2_1^{\mathcal{I}_L} \quad (19)$$

$$R_l = \sum_{k\in\mathcal{I}_{L-1}} r'_{k,l} + \sum_{\substack{\mathcal{A}\in 2_1^{\mathcal{I}_L},\\ l\in\mathcal{A}, k\in\mathcal{I}_{|\mathcal{A}|}}} r'_{\mathcal{A},k,l}, \quad \forall l \in \mathcal{I}_L \quad (20)$$

for some joint distribution, decoding functions and rates satisfying the following

$$H(U_{\mathcal{K},\mathcal{L}}, V_{\mathcal{A}\in\mathcal{T},\mathcal{K}_{\mathcal{A}}}|X) \geq \sum_{\mathcal{A}\in\mathcal{T},k\in\mathcal{K}_{\mathcal{A}}} H(V_{\mathcal{A},k}) - r_{\mathcal{A},k} + \sum_{k\in\mathcal{K},l\in l_{\mathcal{L}}} H(U_{k,l}) - r_{k,l} \quad (21)$$

for any $\mathcal{T} \in 2^{2_2^{\mathcal{I}_L}}$, $\mathcal{K}_{\mathcal{A}} \in 2_1^{\mathcal{I}_L}$ *and* $\mathcal{L} \in 2_1^{\mathcal{I}_L}$ *and*

$$H(U_{\mathcal{I}_{\Xi},\mathcal{A}}, V_{\mathcal{K}\in 2_1^{\mathcal{A}},\mathcal{I}_{|\mathcal{A}\cap\mathcal{K}|}}) \leq \sum_{k\in\mathcal{A},l\in\mathcal{I}_{\Xi}} H(U_{k,l}) + r'_{k,l} - r_{k,l} + \sum_{\mathcal{K}\in 2_1^{\mathcal{A}},K\in\mathcal{I}_{|\mathcal{A}\cap\mathcal{K}|},l\in\mathcal{A}} H(V_{\mathcal{K},k}) + r'_{\mathcal{K},k,l} - r_{\mathcal{K},k} \quad (22)$$

for any $\mathcal{A} \in 2_1^{\mathcal{I}_L}$ *where* $\Xi = \min\{|\mathcal{A}|, L-1\}$.

Base layer codebook generation: For every $\mathcal{A} \in 2_2^{\mathcal{I}_L}$, we generate a set of $|\mathcal{G}|$ MDS codes, where the k^{th} code is a $(|\mathcal{G}|, k)$ MDS code, which is basically a codebook generated with the distribution $P_{V_{\mathcal{A}}}$, and the codeword from this codebook will be binned $|\mathcal{G}|$ times independently.

Refinement layer codebook generation: We generate a set of $L-1$ codes where the k^{th} code is an (L, k) SCEC, which itself consists of a set of L codebooks, one for each channel.

Binning: For each base codebook $C_{\mathcal{A},k}$, we randomly bin the codewords L independent times for each description, with the binning rate $r'_{\mathcal{A},k,l}$ for all $\mathcal{A} \in 2_1^{\mathcal{I}_L}, k \in \mathcal{I}_{|\mathcal{A}|}, l \in \mathcal{I}_L$.

For the refinement codebooks, $C'_{k,l}$, we randomly bin (once), with the binning rate $r'_{k,l}$, for all $k \in \mathcal{I}_{L-1}, l \in \mathcal{I}_L$.

Encoding: Given x^n, the encoder finds two sets of indices $q_{\mathcal{A}\in 2_1^{\mathcal{I}_L},\mathcal{I}_{|\mathcal{A}|}}$ and $q_{\mathcal{I}_{L-1},\mathcal{I}_L}$ such that the codewords associated with these indices are jointly typical.

Decoding: Decoder $\mathcal{G}$ receives the descriptions associated with $\mathcal{G}$ and decodes using the bin indices and the associated base layer.

Corollary 1. $\mathcal{R}_{PPR1} \subseteq \mathcal{R}_{CMS-RB}$, $\mathcal{R}_{PPR2} \subseteq \mathcal{R}_{CMS-RB}$.

Proof: Since $\mathcal{R}_{PPR1} \subseteq \mathcal{R}_{PPR2}$ by construction, we will only show $\mathcal{R}_{PPR2} \subseteq \mathcal{R}_{CMS-RB}$. Setting $V_{\mathcal{A},k,l} = \Phi$ in $\mathcal{RD}_{CMS-RB}$ for $\mathcal{A} \in 2^{\mathcal{I}_L}, k \in \mathcal{I}_{|\mathcal{A}|}, l \in \mathcal{I}_L$, where Φ is deterministic, and setting codebook and binning rates at the refinement layer same value for each descriptions, i.e., $r_{k,l} = r_k, r'_{k,l} = r'_k, \forall l \in \mathcal{I}_L$, yield $\mathcal{R}_{PPR2}$ by construction. ■

IV. Discussion

In this paper, we studied the use of random binning in multiple descriptions coding. We showed that for the two descriptions problem, the currently largest known achievable region obtained with conditional codebook encoding can be achieved with a basic random binning scheme. This result, in addition to being theoretically interesting in itself, has potential practical implications on the design of multiple description codes. We next extended our approach to $L > 2$ descriptions. We presented two coding schemes, first a basic extension of the above two descriptions method without refinement layers, and a more generalized scheme which performs layered random binning. The generalized scheme is intuitively expected to offer benefits whenever there is symmetry within a subset of rates. As part of future work, we will consider the following questions. Can random binning achieve points outside ZB for two the descriptions problem and how do the proposed schemes compare with conditional codebook methods for $L > 2$?

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