# LOCALLY OPTIMAL CODEBOOK DESIGN FOR QUADTREE-BASED VECTOR QUANTIZATION

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## ABSTRACT

The optimal design of quadtree-based vector quantizers is addressed. Until now, work in this area has focused on optimizing the quadtree structure for a given set of leaf quantizers with little attention spent on the design of the quantizers themselves. In cases where the leaf quantizers were considered, codebooks were optimized without regard to the ultimate quadtree segmentation. However, it is not sufficient to consider each problem independently, as separate optimization leads to an overall suboptimal solution. Rather, joint design of the quadtree structure and the leaf codebooks must be considered for overall optimality. The method we suggest is a "quadtree" constrained version of the entropy-constrained vector quantization design method. To this end, a centroid condition for the leaf codebooks is derived that represents a necessary optimality condition for variable-rate quadtree coding. This condition, when iterated with the optimal quadtree segmentation strategy of Sullivan and Baker results in a monotonically descending rate-distortion cost function, and consequently, an (at least locally) optimal quadtree solution.

## 1. INTRODUCTION

Natural images typically consist of regions with widely varying content and activity that often frustrate coding efforts. Because of these nonstationarities, it is desirable to find a segmentation that allocates less bits to homogeneous neighborhoods and more bits to areas containing edges and texture. When this strategy is adopted, the segmentation structure along with the quantization information must be specified to the decoder in order to reconstruct the coded image. One widely used and successful segmentation scheme that can be easily transmitted due to its compact nature is the quadtree data structure [1].

The quadtree data structure is a method for hierarchically decomposing an image into distinct, nonoverlapping regions of varying dimension. For example, an image block of size $M \cdot 2^L \times M \cdot 2^L$ can be decomposed into an $L$ level hierarchy ranging from $4^L$ leaf nodes of size $M \times M$ to a single leaf of dimension $M \cdot 2^L \times M \cdot 2^L$ (see Figure 2). When used for coding, an image is quantized using a set of codebooks matched to the dimensions of the leaf nodes. Until recently, the determination of the quadtree decomposition has been limited to methods that are heuristic in nature [2, 3, 4, 5]. For example, in the *top-down* (splitting) approach [3], starting from the largest possible block, a predefined rule is employed to determine whether a given block should be quantized using a single quantizer, or whether it should be decomposed into four smaller subblocks to be quantized independently. In contrast, the *bottom-up* (merging) construction [2, 4, 5] begins with the smallest possible blocks. Using a predefined criteria, subsequent judgments are made whether any adjacent four subblocks should be merged into a single block to be quantized instead—given that all four subblocks have been previously merged.

While both of these techniques are intuitively appealing, they do *not* determine the optimal quadtree structure in the sense that overall distortion is minimized subject to a constraint on the overall rate. This difficulty was recently surmounted by Sullivan and Baker [6] in which a Lagrangian formulation similar to that of the generalized BFOS algorithm [7] was employed to determine the optimal structure for a given set of quantizers—including the overhead information to specify the tree. Using the nested nature of the quadtree segmentation, this technique eliminates the necessity of exhaustively searching over all possible quadtree structures.

However, the optimality of this approach is limited by the quality of the leaf codebooks. In [6], as with most previous methods, codebooks for each of the varying dimensions are designed without regard for their role as part of the overall quadtree structure. For cases in which an attempt is made to reflect the role of the codebooks, they are heuristic and consequently suboptimal [3]. For the best results, leaf codebooks should reflect the class of the image to which they are called on to represent. Intuitively (and as evidenced by experimental results), we expect that larger block-size codebooks should represent smooth sections, while codebooks for smaller blocks should be targeted for high activity regions involving edges and texture. In any event, for overall optimality, joint design of the quadtree structure and the leaf codebooks must be conducted. To this end, we propose an iterative procedure in Sections 2 and 3 for the design of variable block-size quantizers (and their respective variable length codes) that produces a lo-

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