CODING OF SPECTRAL MAGNITUDES USING OPTIMIZED LINEAR TRANSFORMATIONS

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ABSTRACT

This paper introduces a novel vector quantization (VQ) technique, wherein the quantized vector is obtained by applying a linear transformation selected from a first codebook to a codevector selected from a second codebook. The transformation is selected from a family of linear transformations, represented by a matrix codebook. Vectors in the second codebook are called residual codevectors. In order to avoid high complexity during the search for the best linear transformation, each linear transformation is assigned a representative vector, such that the search can be done employing the representative vectors. The design algorithm is based on joint optimization of the linear transformation and the residual codebooks. It is shown that the proposed technique yields high quality spectral magnitude quantizer with performance exceeding that of multistage vector quantizer (MSVQ) of similar complexity and bit rate.

1. INTRODUCTION

High quality spectral magnitude quantization is crucial to many low bit rate speech coders employing a sinusoidal model [1] [2] [3] [4]. The spectral magnitudes are obtained by sampling the spectrum of either the speech or the LP residual at frequencies corresponding to pitch harmonics. This procedure generates a variable dimension vector since the number of pitch harmonics changes in time.

In this paper, the variable dimension spectral vector is first transformed into a fixed dimension vector, and then the fixed dimension vector is quantized efficiently using the proposed VQ technique. The fixed dimension is chosen such that there is no modeling distortion caused by transformation. The proposed VQ approach reconstructs the input vector by applying a linear transformation selected from a first codebook to a codevector selected from a second codebook. The transformation is selected from a family of linear transformations, represented by codebook of matrices. Vectors in the second codebook are called residual codevectors. In order to avoid high complexity during the search for the best linear transformation, each linear transformation is assigned a representative vector, such that the search can be done employing the representative vectors. The design algorithm is based on joint optimization of the linear transformation and the residual codebooks. It is shown that the proposed technique yields high quality spectral magnitude quantizer with performance exceeding that of multistage vector quantizer (MSVQ) of similar complexity and bit rate.

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2. PROBLEM FORMULATION

Let $\mathbf{x}$ be an M-dimensional input vector. According to the proposed approach, the quantized vector $\hat{\mathbf{x}}$ is given by

$$\hat{\mathbf{x}} = \hat{\mathbf{T}}\hat{\mathbf{c}} \tag{1}$$

where $\hat{\mathbf{T}}$ represents the linear transformation matrix selected from the matrix codebook $\mathcal{C}_T$ and $\hat{\mathbf{c}}$ represents a residual codevector which is a member of the residual codebook $\mathcal{C}_r$.

Using the mean square error (MSE) distortion measure, the average distortion D on the set of vectors $\{\mathbf{x_k}\}$ of size N is

$$D = \frac{1}{N}\sum_{k=0}^{N-1} \|\mathbf{x_k} - \hat{\mathbf{x}}_\mathbf{k}\|^2 \tag{2}$$

or, based on (1):

$$D = \frac{1}{N}\sum_{k=0}^{N-1} \|\mathbf{x_k} - \hat{\mathbf{T}}_\mathbf{k}\hat{\mathbf{c}}_\mathbf{k}\|^2 \tag{3}$$

where $\hat{\mathbf{T}}_\mathbf{k}$ is the transformation matrix and $\hat{\mathbf{c}}_\mathbf{k}$ is the residual codevector corresponding to the input vector $\mathbf{x_k}$.

In this work, the variable dimension spectral magnitudes are transformed into a fixed dimension (M=48) input vectors, using discrete cosine transform (DCT). The objective here is to design the codebooks $\mathcal{C}_T$, $\mathcal{C}_r$ that minimize (3) and to develop an efficient coding rule for this VQ technique.

2.1. Encoding/Decoding

Given the linear transformation codebook $\mathcal{C}_T$ and the residual codebook $\mathcal{C}_r$, the optimal pair $(\hat{\mathbf{T}}, \hat{\mathbf{c}})$ for encoding the vector $\mathbf{x}$ is simply given by

$$(\hat{\mathbf{T}}, \hat{\mathbf{c}}) = \arg \min_{\hat{\mathbf{T}} \in \mathcal{C}_T, \hat{\mathbf{c}} \in \mathcal{C}_r} \|\mathbf{x} - \hat{\mathbf{T}}\hat{\mathbf{c}}\|^2 \tag{4}$$

The minimization required in (4) is computationally intensive if an exhaustive search in both codebooks is employed. To avoid high search complexity, a sequential search is employed whereby the linear transformation $\mathbf{T}$ is determined first.

To simplify the search of the linear transformation codebook $\mathcal{C}_T$, we map this codebook into a set of codevectors $\{\mathbf{t_j}\}$ stored in $\mathcal{C}_t$, so that the ith matrix of $\mathcal{C}_T$ namely, $\mathbf{T_i}$, is mapped into a corresponding codevector, $\mathbf{t_i}$ in $\mathcal{C}_t$. The codebooks $\mathcal{C}_T$ and $\mathcal{C}_t$ are related such that the linear transformation to be assigned to the input vector $\mathbf{x}$ will be given by the code-matrix i iff

$$\mathbf{t_i} = \arg \min_{\mathbf{t_j} \in \mathcal{C}_t} \|\mathbf{x} - \mathbf{t_j}\|^2 \tag{5}$$

Note that the search in (5) has the same computational complexity as the usual VQ search. However, the use of transforms associated with the vectors $\{\mathbf{t_j}\}$ allow us to trade-off a larger memory (required for storing the transforms) for improved performance as it will be shown below.

Once the vector $\mathbf{t_i}$ is determined, the associated linear transformation $\hat{\mathbf{T}} = \mathbf{T_i}$ is employed to search the second stage by choosing $\hat{\mathbf{c}}$ to minimize

$$\min_{\hat{\mathbf{c}} \in \mathcal{C}_r} \|\mathbf{x} - \hat{\mathbf{T}}\hat{\mathbf{c}}\|^2 \tag{6}$$

The quantized vector is given by $\hat{\mathbf{x}} = \hat{\mathbf{T}}\hat{\mathbf{c}}$. Depending on the memory and complexity requirements the search in (6) can be done by either generating the reconstruction vectors using matrix multiplication at the time of search, or storing pre-computed reconstruction vectors. In the former case, the complexity is larger than MSVQ, while in the latter case the complexity is practically the same as in MSVQ.

3. JOINT CODEBOOK OPTIMIZATION

In order to jointly optimize the codebooks, we use an iterative sequential optimization. The algorithm iterates between optimizing linear transformation codebook $\mathcal{C}_T$ and the associated $\mathcal{C}_t$ for a given residual codebook $\mathcal{C}_r$ and optimizing the residual codebook for the given linear transformation codebook.

In order to sequentially optimize the codebooks, the input vector space is partitioned with respect to the codebook whose entries are being optimized. Let $R_{i,j}$ denote the set of input vectors whose assigned indices are i for the codebook $\mathcal{C}_T(\mathcal{C}_t)$, and j for the codebook $\mathcal{C}_r$. Given $R_{i,j}$, the set of input vectors assigned to the ith entry of the codebook $\mathcal{C}_T(\mathcal{C}_t)$ is given by

$$U_i = \bigcup_{j=1}^{N_r} R_{i,j} \tag{7}$$

and the set of vectors assigned to the jth entry of residual codebook $\mathcal{C}_r$ is

$$V_j = \bigcup_{i=1}^{N_T} R_{i,j} \tag{8}$$

where N_r is the size of $\mathcal{C}_r$ and N_T is the size of both $\mathcal{C}_T$ and $\mathcal{C}_t$.

3.1. Design of the Linear Transformation Codebook For a Given Residual Codebook

Given the fixed residual codebook and the partition U_i, our objective is to compute $\mathbf{T_i}$ for $i = 1, \ldots, N_T$ to minimize (3). In other words, $\mathbf{T_i}$ is obtained as the solution of the optimization problem

$$\mathbf{T_i} = \arg \min_{\hat{\mathbf{T}}} \sum_{k:\mathbf{x_k} \in U_i} \|\mathbf{x_k} - \hat{\mathbf{T}}\hat{\mathbf{c}}_\mathbf{k}\|^2 \tag{9}$$

The solution of the above minimization problem may not be unique. The ith centroid will be chosen as the solution with the minimum Frobenius norm ($\sqrt{\text{trace}[\hat{\mathbf{T}}'\hat{\mathbf{T}}]}$) and is given by

$$\mathbf{T_i} = \mathbf{X}\mathbf{Y}^+ \tag{10}$$

where $\mathbf{X}$ and $\mathbf{Y}$ are matrices that have $\{\mathbf{x_k}\}_{k=1}^{k=\|U_i\|}$ and $\{\hat{\mathbf{c}}_\mathbf{k}\}_{k=1}^{k=\|U_i\|}$ as their columns respectively. $\mathbf{Y}^+$ denotes the pseudoinverse of $\mathbf{Y}$, and $\|U_i\|$ denotes the cardinality of U_i.

Experimental evidence shows that a good way of designing $\mathbf{t_i}$ for $i = 1, \ldots, N_T$, is to update $\mathbf{t_i}$ as the Euclidean centroid of the reconstructed vectors $\hat{\mathbf{x}}_\mathbf{k}$ whose input vectors $\mathbf{x_k} \in \mathbf{U_i}$;

$$\mathbf{t_i} = \frac{1}{\|U_i\|} \sum_{k:\mathbf{x_k} \in U_i} \hat{\mathbf{x}}_\mathbf{k} = \frac{1}{\|U_i\|} \sum_{k:\mathbf{x_k} \in U_i} \mathbf{T_i}\hat{\mathbf{c}}_\mathbf{k} \tag{11}$$

There is a simple analytical justification for this approach. In the case of high bit rate quantization or highly clustered input vectors, for an input vector $\mathbf{x_n}$ which has $\mathbf{t_i} = \arg\min_{\mathbf{t_j} \in \mathcal{C}_t} \|\mathbf{x_n} - \mathbf{t_j}\|^2$, the Euclidean distance between $\mathbf{t_i}$ and $\hat{\mathbf{x}}_\mathbf{n}$ will be small due to (11). Furthermore using the triangle inequality the Euclidean distance between $\mathbf{x_n}$ and $\hat{\mathbf{x}}_\mathbf{n}$ can be upper bounded as

$$\|\mathbf{x_n} - \hat{\mathbf{x}}_\mathbf{n}\| \leq \|\mathbf{x_n} - \mathbf{t_j}\| + \|\mathbf{t_j} - \hat{\mathbf{x}}_\mathbf{n}\| \tag{12}$$

The right hand side of (12) is expected to have a low value at $j = i$, because the first term is minimized by the choice $j = i$ and the second term corresponds to the distance between a vector and its centroid. This shows that by employing the sequential encoding rule given by (5) we can obtain a low value for the upper bound on the quantization error.

3.2. Design of the Residual Codebook For a Given Linear Transformation Codebook

Given the fixed linear transformation codebook and the partition V_j, we will compute $\mathbf{c_j}$ for $j = 1, \ldots, N_r$ to minimize (3). So $\mathbf{c_j}$ will be given by

$$\mathbf{c_j} = \arg \min_{\hat{\mathbf{c}}} \sum_{k:\mathbf{x_k} \in V_j} \|\mathbf{x_k} - \hat{\mathbf{T}}_\mathbf{k}\hat{\mathbf{c}}\|^2 \tag{13}$$

The minimum norm solution of the above minimization is the centroid equation for the jth centroid and computed as

$$\mathbf{c_j} = \mathbf{A}^+\mathbf{b} \tag{14}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{T_1} \\ \vdots \\ \mathbf{T}_{\|\mathbf{V_j}\|} \end{bmatrix} \mathbf{b} = \begin{bmatrix} \mathbf{x_1} \\ \vdots \\ \mathbf{x}_{\|\mathbf{V_j}\|} \end{bmatrix} \tag{15}$$

3.3. Joint Codebook Design

The main design algorithm can now be stated by using the centroid computations and the sequential encoding rule described in earlier sections. The initialization starts with the codebook $\mathcal{C}_t$ which is initialized using the codebook splitting method on the training vectors $\{\mathbf{x_k}\}$. Then $\mathcal{C}_T$ is initialized such that each code-matrix $\mathbf{T_i}$ is an orthogonal matrix, and $\mathbf{T}'_\mathbf{i}\mathbf{t_i}$ is a constant vector for all i. The reason for this is to have the training vectors for the residual codebook $\mathcal{C}_r$, namely $\{\mathbf{T}'_\mathbf{k}\mathbf{x_k}\}$, mainly distributed around this constant vector, thereby decreasing the volume of the region spanned by the training vectors that are assigned to a given code-matrix. Using this new set of vectors, $\{\mathbf{T}'_\mathbf{k}\mathbf{x_k}\}$, the residual codebook $\mathcal{C}_r$ is designed by codebook splitting.

Once the codebooks are initialized, the main design algorithm performs the following steps:

1. Partition the training set to obtain $R_{i,j}$.
2. Compute the overall distortion, if termination criterion is satisfied then stop else continue.
3. Compute the optimum codebook $\mathcal{C}_T$ using (10), update the codebook $\mathcal{C}_t$ using (11).
4. Partition the training set to obtain a new $R_{i,j}$.
5. Compute the optimum codebook $\mathcal{C}_r$ using (14).
6. Go to 1.

While steps 3 and 5 of this algorithm always decrease the overall distortion, the partitioning steps 1 and 4 may increase the distortion due to the suboptimal sequential encoding rule. Hence, the algorithm does not guarantee strict descent, however, in practice the distortion generally decreases. The termination criterion adopted in this algorithm is to stop when the relative change in the distortion is less than a given threshold.

4. EXPERIMENTAL RESULTS

In order to evaluate the performance of the proposed VQ technique for spectral magnitude quantization, we compared this technique with a two stage MSVQ. The speech material used is sampled at 8kHz and consists of sentences spoken by male and female speakers. The spectral vectors are extracted every 10 ms and normalized by a gain factor. A set of 88064 vectors are used for training, and another set of 22016 vectors are used for testing. All the vectors have dimension 48 or smaller. Thus the distortion incurred consists of quantization distortion only, since the modeling distortion is zero. The objective performance is measured by using root square spectral distortion (SD) in dB, which is defined as

$$SD = \frac{1}{N}\sum_{k=0}^{N-1}\sqrt{\frac{1}{L_k}\sum_{m=0}^{L_k-1}(10\log_{10}\frac{s_k^2[m]}{\hat{s}_k^2[m]})^2} \qquad (16)$$

where L_k is the dimension of the spectral magnitude vector $\mathbf{s_k}$ and $s_k[m]$ denotes the mth element of the vector $\mathbf{s_k}$.

The performance of the proposed and the multi-stage VQ over the training set measured in terms of SD, and the percentage of vectors with distortion exceeding 5 dB and 7 dB is shown in Table 1. Table 2 shows the performance over the test set. The design is done for various typical bit rates used in low bit rate speech coders. As shown, the proposed technique has a better SD, and less outliers than the two-stage MSVQ.

Rate	SD		5dB and 7dB outliers (%)			
(bits)	Proposed	MSVQ	Proposed		MSVQ	
10=5+5	3.53	3.92	11.37	3.65	16.66	5.29
11=5+6	3.39	3.82	9.93	3.34	15.51	4.92
11=6+5	3.32	3.78	9.32	3.07	14.99	4.71
12=6+6	3.14	3.70	7.80	2.67	14.06	4.37
13=6+7	3.07	3.62	7.53	2.36	13.24	4.40
13=7+6	2.86	3.59	6.31	1.93	12.76	4.14
14=7+7	2.79	3.51	5.85	1.87	11.94	3.90

Table 1: Design performance

Rate	SD		5dB and 7dB outliers (%)			
(bits)	Proposed	MSVQ	Proposed		MSVQ	
10=5+5	3.55	3.92	11.47	3.72	16.80	5.23
11=5+6	3.42	3.82	10.10	3.47	15.35	5.09
11=6+5	3.37	3.79	9.77	3.22	15.08	5.01
12=6+6	3.18	3.70	8.10	2.79	13.93	4.46
13=6+7	3.12	3.63	7.98	2.43	13.32	4.52
13=7+6	2.95	3.60	6.84	2.19	12.74	4.19
14=7+7	2.87	3.53	6.21	2.12	11.95	3.98

Table 2: Test performance

5. REFERENCES

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