ADAPTIVE STATE ESTIMATION OVER LOSSY SENSOR NETWORKS FULLY ACCOUNTING FOR END-TO-END DISTORTION

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ABSTRACT

This paper investigates state estimation with wireless sensors communicating over unreliable bandwidth-limited networks. Specifically, we consider so-called smart sensors equipped with simple processing units, which enable predictive coding at the sensor side, to meet the bandwidth constraint. While predictive coding significantly reduces the bit-rate by removing temporal redundancies, it exacerbates the impact of packet loss, due to error propagation through the prediction loop, potentially causing significant degradation of the reconstructed signal. To fully account for and control the conflict between coding efficiency and robustness to packet loss, we propose a coding approach that explicitly optimizes the tradeoff between rate and end-to-end distortion (EED). The proposed method determines optimal switching decisions between available coding modes, offering different compression-robustness operating points, based on EED that is optimally estimated at the encoder (sensor), in order to realize the best rate-distortion tradeoff. Simulation results demonstrate that the proposed approach achieves considerable gains in signal-to-noise ratio for state estimation over lossy sensor networks.

Index Terms— Kalman filter, wireless sensors, lossy networks, end-to-end distortion, joint source-channel coding

1. INTRODUCTION

Wireless sensor networks, wherein wireless channels are used to communicate between sensors and actuators, have been the subject of growing interest in recent years [1]. While wireless networks eliminate the need for wiring and hence are much easier to deploy in many applications, they also pose several significant challenges. A main challenge lies in the unreliable nature of wireless networks which, when combined with constrained transmitting power due to limited embedded battery life, could result in high packet loss rates and limited channel bandwidth.

Various paradigms for state estimation over unreliable channels have been proposed from a variety of perspectives. In [2], a framework was proposed to capture the effect of channel noise on conventional control systems, coupled with the corresponding stability analysis. Kalman filtering with intermittent observations (i.e., due to packet losses) is studied in [3], where convergence conditions were derived given the observation arrival probability. In [1], different lossy network models were considered and the corresponding optimal estimate was derived along with stability analysis. Moreover, the use of error correcting codes in sensor networks was studied in [4] with respect to its power efficiency.

On another front, to satisfy the data rate constraint, quantization and sampling were introduced into conventional control systems [5].

The minimum rate needed to stabilize different systems through various channels was studied in [6, 7]. To further improve the compression performance, predictive coding was employed in [8], wherein only the sign of the innovation is transmitted, while [9] proposed an extension where the innovation is coded with a multi-level quantizer.

Clearly, there is an inherent conflict between the need to reduce the data rate and the objective of better error control. On the one hand, to achieve robustness to network errors, redundant information is usually needed which increases the rate. On the other hand, to substantially reduce the rate, one may need to employ predictive coding which exacerbates the impact of packet loss on the state estimate due to error propagation through the prediction loop, which could critically compromise performance [10].

Therefore, the tradeoff between compression efficiency and robustness to channel errors is crucial to the problem of state estimation over unreliable channels. To control this tradeoff, [11] proposes two coding modes for smart wireless sensors, where at each time instant the sensors select one of the modes according to a cost function involving the receiver's long-term average estimation error covariance and the transmission energy. To mitigate the excessive computational load due to the mode selection procedure, another solution is also proposed in [11] to provide simpler but suboptimal decisions.

In this paper, a novel approach to account for the tradeoff between rate and state estimation quality is proposed. Instead of the long-term average estimation error, we proposed to consider the end-to-end distortion (EED), which is the distortion between the *encoder estimated state* and the *decoder estimated state*. Since each channel realization is not available to the encoder, we recursively estimate the mean and correlation matrix (averaging over channel realizations, not state realizations) of the EED at the encoder (sensor), leveraging a simple approach that originated in the signal compression field [12], which enables explicit rate-distortion (R-D) optimization to decide the current optimal coding mode for a specific realization of the state process. Simulation results show that the proposed method is capable of providing considerable gains in signal-to-noise ratio of state estimation over lossy networks, given a prescribed rate constraint.

2. PROBLEM FORMULATION

2.1. System setup

Consider the basic state estimation scheme shown in Fig. 1, with a single wireless smart sensor carrying an embedded processing unit. Let the state process be the following auto-regression (AR) process:

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + \mathbf{w}_k, \tag{1}$$
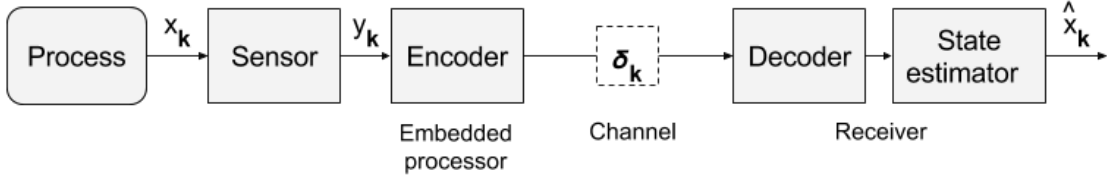
Fig. 1: The basic setup for state estimation with a single wireless smart sensor

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector at time instant k, $A \in \mathbb{R}^{n \times n}$ is the system matrix and $\mathbf{w}_k$ is Gaussian white vector noise with covariance matrix Q.

The measurement process at the sensor is given by:

$$\mathbf{y}_k = C\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{y}_k \in \mathbb{R}^m$ is the observation vector at time instant k, $C \in \mathbb{R}^{m \times n}$ and $\mathbf{v}_k$ is Gaussian white vector noise with covariance matrix R.

At each time instant, the sensor generates an observation $\mathbf{y}_k$ and the processor chooses from available coding modes, denoted by t_k (details of these modes will be discussed in section 3), to encode the information and transmit the packet via a lossy channel.

In this paper we consider channels with packet loss, where the model is given by a random process δ_k. $\delta_k = 0$ indicates a lost packet at time instant k, and $\delta_k = 1$ otherwise. In the setting we consider, the encoder has no feedback from the channel, and thus has no access to δ_k. It only knows the channel statistics. The receiver, on the other hand, has direct access to δ_k. For simplicity, it is assumed that δ_k is an independent and identically distributed (i.i.d.) process, noting that the proposed approach can easily be extended to more complex packet loss models. The packet loss rate is denoted as p.

After decoding, the receiver calculates and outputs its state estimate $\hat{\mathbf{x}}_k$. Approaches to calculate $\hat{\mathbf{x}}_k$ naturally depend on the coding mode of the current packet, and whether or not it was received, as will be detailed in section 3.

2.2. Target cost function based on EED

At the encoder, different coding modes provide different tradeoffs between compression efficiency and its robustness to channel loss. How the encoder accounts for this tradeoff and determines the best mode at each time instant, given the sensor observations, is our main concern in this paper.

From a compression perspective, this tradeoff is equivalent to achieving the best state estimation performance over a lossy network given a target rate, and is a constrained optimization problem often referred to as rate-distortion (R-D) optimization. Thus we propose to select the coding mode t_k by minimizing a Lagrangian cost function, i.e., $t_k = \arg\min_\tau J(\tau)$ where,

$$J(\tau) = E\{D_e(\tau)\} + \lambda Rate(\tau), \tag{3}$$

where $Rate(\tau)$ is the bitrate needed for mode τ, and λ is the Lagrangian parameter. D_e is the end-to-end distortion (EED), which is defined as the distortion between the receiver-end state estimate $\hat{\mathbf{x}}_k$ and the sensor-end state estimate $\bar{\mathbf{x}}_k$. Since the encoder does not know which packets were lost it does not know D_e exactly, hence it instead uses its best estimate, its expected value, as will shortly be explained in detail. Here, $\bar{\mathbf{x}}_k$ is given by a local steady-state Kalman filter operated at the sensor's embedded processing unit:

$$\bar{\mathbf{x}}_k = A\bar{\mathbf{x}}_{k-1} + K_s(\mathbf{y}_k - CA\bar{\mathbf{x}}_{k-1}), \tag{4}$$

where K_s is the steady state Kalman filter gain, given by: $K_s = P_sC^T(CP_sC^T + R)^{-1}$ and P_s is the steady state error covariance which satisfies the Riccati equation: $P_s = AP_sA^T + Q - AP_sC^T(CP_sC^T + R)^{-1}CP_sA^T$.

In (3), D_e is introduced to account for the state estimation error associated with channel loss and quantization error, and is the state estimation mismatch between the encoder and decoder. We adopt the mean square error as criterion: $D_e = ||\bar{\mathbf{x}}_k - \hat{\mathbf{x}}_k||_2^2$. Note that by focusing on D_e instead of the overall $D = ||\mathbf{x}_k - \hat{\mathbf{x}}_k||_2^2$ we are neglecting the estimation error $\mathbf{x}_k - \bar{\mathbf{x}}_k$ introduced at the sensor, before quantization and transmission. We will next show that by making the assumption that the sensor estimation error is uncorrelated with the state estimate mismatch between encoder and decoder, we ensure that $E\{D_e\} = E\{D\} - c$ and the R-D decision based on $E\{D_e\}$ is the same as if it were based on the expected overall state estimation error.

Lemma 1. *If the sensor estimation error $\mathbf{x}_k - \bar{\mathbf{x}}_k$ is uncorrelated with the state estimate mismatch between encoder and decoder $\bar{\mathbf{x}}_k - \hat{\mathbf{x}}_k$, then using D_e in (3) yields the same decision as using the overall state estimation error $D = ||\mathbf{x}_k - \hat{\mathbf{x}}_k||_2^2$.*

Proof. $E\{D\} = E\{||\mathbf{x}_k - \bar{\mathbf{x}}_k||_2^2\} + E\{D_e\} + E\{(\mathbf{x}_k - \bar{\mathbf{x}}_k)^T(\bar{\mathbf{x}}_k - \hat{\mathbf{x}}_k)\}$, where $E\{||\mathbf{x}_k - \bar{\mathbf{x}}_k||_2^2\}$ is a constant c given A, C, Q, R. By the assumption, the last term is 0 since $E\{(\mathbf{x}_k - \bar{\mathbf{x}}_k)\} = \mathbf{0}$. Therefore $E\{D\} = E\{D_e\} + c$ and will produce the same decision to minimize (3). $\square$

From Lemma 1, it follows that if the assumption holds approximately, then EED, while directly accounting for the channel loss for a specific realization, also represents approximately the overall state estimation performance.

Recall that we use $E\{D_e\}$ in (3), rather than the exact D_e. The encoder does not have access to the decoder estimated state $\hat{\mathbf{x}}_k$ and its best recourse is to estimate EED given its knowledge of the channel statistics. The estimation is given by:

$$E\{D_e(\tau)\} = E_c\{||\bar{\mathbf{x}}_k - \hat{\mathbf{x}}_k||_2^2 \mid t_k = \tau\} \tag{5}$$

where we use the streamlined notation $E_c\{\cdot\}$ to represent expectation conditioning on past observations and coding modes, i.e., $E_c\{\cdot\} = E\{\cdot \mid t_i, \mathbf{y}_j : i = 0, 1, ..., k - 1; j = 0, 1, ..., k\}$. Since the estimation is conditioned on the sensor observations, it is still able to capture the effects of channel loss for a specific measurement from the sensor.

Note that calculating $E\{D_e(\tau)\}$ is not a simple task considering the propagation of errors from possible lost packets in the past, through the prediction loop and the Kalman filter recursions. In section 3 we will introduce an EED estimation approach which optimally estimates $E\{D_e(\tau)\}$ with a reasonable complexity via a recursive formula.

3. CODING MODES AND EED ESTIMATION

As presented in section 2, the encoder needs to obtain an EED estimate for each mode to determine the optimal via (3). However, due to the prediction loop introduced by some encoding modes, as well as the recursive nature of Kalman filters, estimating EED in (5) is not straightforward, since the number of possible error propagation scenarios increases exponentially with the number of transmitted packets. Furthermore, a Monte Carlo simulation at the sensor is impractical due to its excessive computational load.

In this section, we introduce the encoding and decoding procedure for each mode, as well as a recursive technique to optimally estimate EED at low to moderate complexity.

3.1. Observation mode

We first introduce the prevalent "observation" mode ($t_k = 0$), where the quantized observation $\mathbf{y}_k^q$ is sent to the decoder. The decoder generates its receiver-end state estimate by:

$$\hat{\mathbf{x}}_k = A\hat{\mathbf{x}}_{k-1} + \delta_k K_d(\mathbf{y}_k^q - CA\hat{\mathbf{x}}_{k-1}), \tag{6}$$

which is a steady-state Kalman filter with intermittent observations, where $K_d = P_d C^T (CP_d CC^T + R)^{-1} CP_d A^T$ and P_d satisfies the modified Riccati equation [13]: $P_d = AP_d A^T + Q - (1 - p)AP_d C^T (CP_d C^T + R)^{-1} CP_d A^T$. The steady state Kalman filter accounts for the channel loss and ignores the quantization noise since it is assumed to be dominated by the observation noise and channel errors.

At the encoder, the EED estimation is given by:

$$E_c\{||\bar{\mathbf{x}}_k - \hat{\mathbf{x}}_k||_2^2\} = \bar{\mathbf{x}}_k^T \bar{\mathbf{x}}_k - 2\bar{\mathbf{x}}_k^T E_c\{\hat{\mathbf{x}}_k\} + Tr(E_c\{\hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^T\}), \tag{7}$$

where $Tr(\cdot)$ denotes the trace. Note that the sensor state estimate $\bar{\mathbf{x}}_k$ is known to the encoder and is not considered random for the estimation.

It is evident from (7) that the encoder only needs to obtain the first moment $E_c\{\hat{\mathbf{x}}_k\}$ and the matrix of second moments $E_c\{\hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^T\}$ (equivalently the mean vector and covariance matrix of $\hat{\mathbf{x}}_k$).

For the observation mode ($t_k = 0$), we derive the following recursive formulas to calculate the first and second moments given the known decoder procedure (6):

$$E_c\{\hat{\mathbf{x}}_k\} = AE_c\{\hat{\mathbf{x}}_{k-1}\} + (1-p)K_d(\mathbf{y}_k^q - CAE_c\{\hat{\mathbf{x}}_{k-1}\});$$
$$E_c\{\hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^T\} = pAE_c\{\hat{\mathbf{x}}_{k-1}\hat{\mathbf{x}}_{k-1}^T\}A^T + (1-p)$$
$$(FE_c\{\hat{\mathbf{x}}_{k-1}\hat{\mathbf{x}}_{k-1}^T\}F^T + K_d\mathbf{y}_k^q E_c\{\hat{\mathbf{x}}_{k-1}^T\}F^T +$$
$$FE_c\{\hat{\mathbf{x}}_{k-1}\}(\mathbf{y}_k^q)^T K_d^T + K_d\mathbf{y}_k^q(\mathbf{y}_k^q)^T K_d^T), \tag{8}$$

where $F = A - K_d CA$.

As can be seen from (8), *only the moments from the previous time instant* ($k-1$) are needed to calculate the moments at time k. Therefore, via the recursive update of (8), we can track the moments of $\hat{\mathbf{x}}_k$ and hence optimally estimate the EED associated with the observation mode.

3.2. Innovation mode

To achieve a better compression performance, in the innovation mode ($t_k = 1$), we choose to send the quantized innovation $\mathbf{z}_k^q$, where the innovation is $\mathbf{z}_k = \mathbf{y}_k - CA\tilde{\mathbf{x}}_{k-1}$, where $\tilde{\mathbf{x}}_k$ is the receiver-end state estimation at no channel loss, which can also be

obtained at the sensor. The decoder generates the receiver-end state estimate by:

$$\hat{\mathbf{x}}_k = A\hat{\mathbf{x}}_{k-1} + \delta_k K_d \mathbf{z}_k^q. \tag{9}$$

Ideally, the innovation should contain all the new information from the observation $\mathbf{y}_k$. Therefore, with proper coding, the innovation mode requires a much lower rate. However, in the presence of channel loss, the encoder and decoder may "drift apart" and have different prior estimated states. This may severely compromise the usefulness of the innovation information. Hence, this mode is more heavily impacted by channel loss. The introduction of this mode largely targets the rate side of the tradeoff, and it is crucial to be able to estimate the EED to judiciously activate this mode at the encoder.

Similar to (8), we also establish the EED estimation procedure by recursively estimating the first and second moments of $\hat{\mathbf{x}}_k$ for the innovation mode:

$$E_c\{\hat{\mathbf{x}}_k\} = AE_c\{\hat{\mathbf{x}}_{k-1}\} + (1-p)K_d\mathbf{z}_k^q;$$
$$E_c\{\hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^T\} = AE_c\{\hat{\mathbf{x}}_{k-1}\hat{\mathbf{x}}_{k-1}^T\}A^T +$$
$$(1-p)(K_d\mathbf{z}_k^q E_c\{\hat{\mathbf{x}}_{k-1}^T\}A^T +$$
$$AE_c\{\hat{\mathbf{x}}_{k-1}\}(\mathbf{z}_k^q)^T K_d^T + K_d\mathbf{z}_k^q(\mathbf{z}_k^q)^T K_d^T), \tag{10}$$

Here too the updates only require the first and second moments of the previous receiver-end state estimate $\hat{\mathbf{x}}_{k-1}$, enabling an efficient EED estimation for the innovation mode.

3.3. Sensing state mode

While the innovation mode is particularly sensitive to channel loss, the observation mode also suffers from error propagation due to the recursive nature of Kalman filters. To introduce a "full reset" of the mismatch between the sensor and receiver, the sensing state mode is included, where the sensor-end state estimation $\bar{\mathbf{x}}_k$ (sensing state) is quantized and sent to the decoder. The decoder simply uses the quantized sensing state to reset its state estimation, if the packet is received:

$$\hat{\mathbf{x}}_k = \delta_k \bar{\mathbf{x}}_k^q + (1 - \delta_k)A\hat{\mathbf{x}}_{k-1}. \tag{11}$$

The recursive EED estimation is straightforward:

$$E_c\{\hat{\mathbf{x}}_k\} = (1-p)\bar{\mathbf{x}}_k^q + pAE_c\{\hat{\mathbf{x}}_{k-1}\};$$
$$E_c\{\hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^T\} = (1-p)\bar{\mathbf{x}}_k^q(\bar{\mathbf{x}}_k^q)^T + pAE_c\{\hat{\mathbf{x}}_{k-1}\hat{\mathbf{x}}_{k-1}^T\}A^T, \tag{12}$$

which again only requires the first and second moments of $\hat{\mathbf{x}}_{k-1}$.

The sensing state mode stops all error propagation from previous state estimation, and thus is most robust to channel loss. However, coding the state estimation directly often requires a significantly higher rate.

In summary, the three modes introduced in this section offer differing operating points in terms of the tradeoff between robustness to packet loss and data rate. The optimal EED estimation approach proposed here enables the encoder to minimize (3) to determine the optimal coding mode at each time intant.

4. SIMULATION RESULTS

To achieve preliminary proof of concept and evidence for the power of the approach we consider a simple setting where the system, observation and covariance matrices are in fact scalar:

$$A = 0.96, C = 0.6, Q = 0.5, R = 0.05.$$

We assume an i.i.d. packet loss process with loss rate p.

| (a) Observation mode | (b) Innovation mode | (c) Sensing state mode |
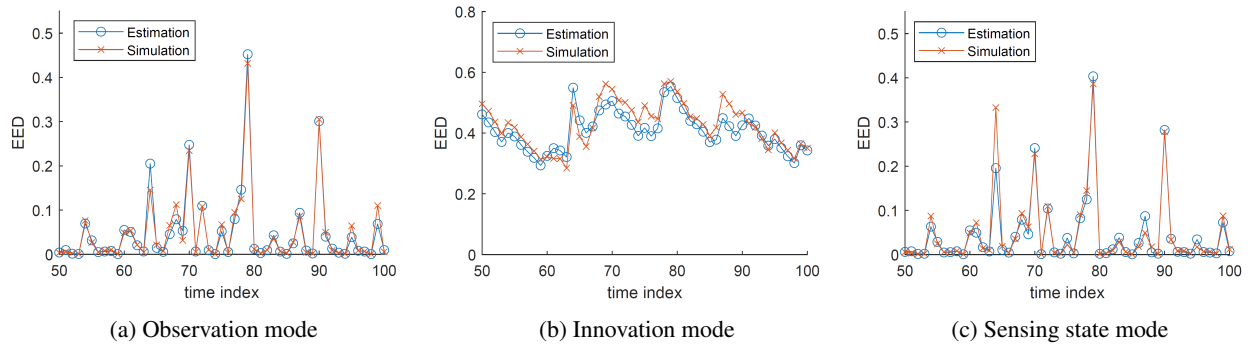
Fig. 2: Comparison of simulated EED with the EED estimation by the proposed method

| (a) Set 1 | (b) Set 2 |
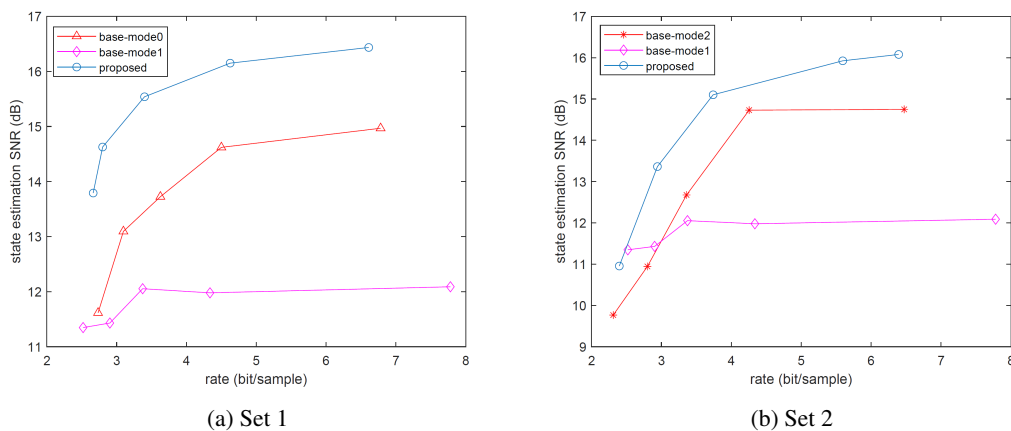
Fig. 3: Rate-SNR plots of the proposed method compared to single mode coding with packet loss rate $p = 0.05$.

Scalar uniform quantizers are designed for each different mode. The quantization levels are entropy coded by Huffman coding according to their probability of occurrence.

First, we evaluate the EED estimation accuracy of the proposed approach. Fig. 2 compares the encoder's EED estimate with actual decoder EED measurements obtained by averaging over 200 random channel realizations. The packet loss rate is set to $p = 0.05$. It is evident that the proposed EED tracking technique provides the encoder with an accurate EED estimate for all three modes, which is critical to enable effective mode selection at the encoder.

Next, the effectiveness of the proposed mode-switching decision based on EED estimation, is presented. Here we consider two sets of comparisons. In set 1, the observation baseline (denoted by base-mode0) is given by using only the observation mode where different quantization intervals are used to generate the rate-SNR curve. The innovation baseline is also presented similarly (base-mode1). The proposed method considers both the observation and innovation modes, and switches between them according to the estimated EED and bitrate needed for each mode (side information to specify the mode is included in the rate calculation). The SNR versus rate curve is obtained by varying the Lagrange parameter λ in (3), where quantization intervals are separately designed for each λ. Set 2 considers a similar comparison, where the sensing state mode replaces the observation mode in both baseline (base-mode2) and the proposed method.

The rate-SNR curves are shown in Fig. 3. Note the SNR here is the signal-to-noise ratio of the ultimate state estimation which considers the actual state estimation distortion, instead of EED. The results reflect averaging over 200 signal realizations and 200 channel realizations with $p = 0.05$.

From the figure, it is evident that the proposed method enables the encoder to effectively switch between two different modes and considerably enhance the ultimate state estimation for a prescribed data rate (∼**2 dB** gain for set 1 and ∼**1 dB** gain for set 2). Moreover, this SNR gain is observed consistently across a wide range of rates, which further proves the reliability and effectiveness of the approach.

It should be emphasized that the experiments with scalar states and observations establish the proof of concept, and similar benefits are expected in the vector case, which is currently under investigation.

5. CONCLUSION

This paper presents a novel approach to adaptively optimize the tradeoff between rate and EED for state estimation in lossy sensor networks. Optimal EED estimation for available coding modes is obtained via a low complexity recursive technique, and enables R-D optimal coding mode decisions. Experimental evidence substantiates the accuracy of the recursive EED estimation technique, and the efficacy of the overall approach, which achieves considerable SNR gains over a wide range of rates.

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