Transmission Energy Allocation with Low Peak-to-Average Ratio

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***Abstract*— Transmission energy allocation (TEA) to bits according to their sensitivity is known to significantly enhance robustness to channel errors. These advantages are gained at the cost of high peak-to-average ratio (PAR) of the signal energy employed to transmit different bits. We show that, in the case of 4-QAM, appropriate grouping of bits allows achieving all the gains of optimal TEA while maintaining PAR at a small fraction of a decibel. Alternatively, we show how to achieve close to optimal TEA under the constraint of perfect (0 dB) PAR, thus extending the application of TEA to constant envelope modulation schemes. Performance is illustrated with an example of Gauss–Markov sources compressed by vector quantization.**

***Index Terms*— Joint source-channel coding, peak-to-average ratio, quadrature amplitude modulation, vector quantization.**

I. Introduction

TRANSMISSION energy allocation (TEA) refers to distributing the available quota of transmission energy to the different bits according to their importance in order to provide optimal unequal error protection. An application of TEA to pulse-coded modulation was described in the early work of Bedrosian [1] and the subsequent work of Sundberg [2]. Recently, it was demonstrated that the robustness of a vector quantizer (VQ) to channel errors can be significantly improved via TEA. The case of a full-search VQ was addressed in [3] and [4], while the extension to multistage VQ was described in [5].

A major drawback of TEA stems from the fact that signal energy employed for transmission of different bits tends to vary considerably. In other words, the peak-to-average ratio (PAR) of the energy of signals transmitted over different modulation intervals is typically high (of the order of 2–3 dB). In this letter we first describe a method to reduce the PAR requirements without performance loss. Subsequently we show that the PAR can be made perfect (0 dB), with a minimal loss in the robust performance. This is particularly interesting since with this modification TEA can be applied to constant envelope modulation schemes. While the proposed ideas are generally applicable, we explain them in the context of VQ based communication.

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TABLE I
Transmission Energy Allocation and PAR in the Case of Two-Dimensional VQ of Codebook Size 256 for Gauss–Markov Source with Correlation Coefficient ρ. TEA Is Optimized for a Channel SNR of 8 dB

	$\rho = 0.9$	$\rho = 0.95$
e_1	10.58	11.44
e_2	9.16	10.43
e_3	7.58	8.10
e_4	7.03	5.90
e_5	5.98	5.19
e_6	4.55	4.20
e_7	3.41	3.09
e_8	2.12	2.14
PAR (dB)	2.25	2.58

TABLE II
Transmission Energy per Two-Dimensional Modulation Interval After Grouping of Bits, and Resulting PAR for The Example of Table I

	$\rho = 0.9$	$\rho = 0.95$
$e_1 + e_8$	12.70	13.58
$e_2 + e_7$	12.57	13.51
$e_3 + e_6$	12.13	12.30
$e_4 + e_5$	13.01	11.09
PAR (dB)	0.13	0.32

II. A Review of TEA For VQ Applications

Consider a source-optimized VQ with codebook $C = \{y_0, y_1, \cdots, y_{2^n-1}\}$. Given source vector x , the nearest neighbor search based VQ encoder transmits an n-bit index I. The index bits are transmitted over a Gaussian channel using binary modulation. The decoder receives a noisy version of the transmitted signals and decodes an index J using hard decision decoding. In other words, the individual bits are transmitted on independent binary symmetric channels. The bit error rate on each of these binary channels depends on the energy used for transmission of the corresponding bit. The decoder produces as estimate y_J—the codevector addressed by the index J in the codebook.

Index assignment involves assigning binary indices to the codewords in order to minimize the distortion resulting from channel errors. Let us employ the following bit explicit notation for the indices $I = (i_1\ i_2\ \cdots\ i_n)$. The sensitivity of the jth bit is defined as the expected amount of distortion caused

TABLE III
PERFORMANCE COMPARISON OF OPTIMAL (Opt) TEA, CONSTRAINED (CONSTR) TEA AND PSEUDO-GRAY CODING (PG) WHICH SHOWS WHAT PORTION OF THE POSSIBLE GAINS OF OPT-TEA OVER PG ARE CAPTURED BY CONSTR-TEA. THE SOURCE AND VQ PARAMETERS ARE THE SAME AS IN TABLE I AND TABLE II. TEA IS OPTIMIZED FOR A CHANNEL SNR (C-SNR) of 8 dB, WHILE THE PERFORMANCE IS EVALUATED FOR A C-SNR OF 4–10 dB

C-SNR	$\rho = 0.9$			$\rho = 0.95$		
dB	Opt-TEA	Constr-TEA	PG	Opt-TEA	Constr-TEA	PG
4	7.13	7.00	5.03	7.56	7.27	4.15
5	9.35	9.22	6.76	9.92	9.63	5.92
6	11.93	11.79	8.85	12.64	12.37	8.06
7	14.93	14.73	11.42	15.69	15.46	10.66
8	17.78	17.65	14.42	18.71	18.55	13.77
9	20.37	20.37	17.78	21.58	21.28	17.46
10	22.46	22.46	21.15	23.51	23.51	21.28

by a bit error at this location:

$$D_j = E||y_{i_1,i_2,\cdots,i_j,\cdots,i_n} - y_{i_1,i_2,\cdots,i_j^*,\cdots,i_n}||^2$$

where $i^* = 1 - i$ denotes the complement of bit i. Neglecting the probability of more than one bit error in a single codevector transmission, the distortion due to channel errors can be written as

$$D_c = \sum_{j=1}^{n} D_j \epsilon_j,$$

where, ϵ_j is the bit error rate for bit j.

The natural binary code (NBC) is a form of index assignment that is obtained when the splitting initialization is employed to design the VQ (see, e.g., [6]). An important consequence of employing NBC for VQ indexing is that it produces a large variation in bit sensitivities. We exploit this feature by providing optimal unequal error protection to the various bits. This is achieved as follows. Let e_j denote the transmission energy allocated for binary modulation of bit i_j. If we denote the representative level of Gaussian noise in the channel by σ_r^2, then the bit error rate ϵ_j is given by $\epsilon_j = \text{erfc}\left(\sqrt{e_j/\sigma_r^2}\right)$. Our aim is to minimize D_c through optimal allocation of transmission energies $\{e_j\}$ subject to the constraint $\sum_{j=1}^{n} e_j = e_{\text{tot}}$, where e_{tot} is the total energy available for transmission of the n bits. A simple and fast energy allocation algorithm was proposed in [3].

Although the technique of TEA achieves impressive performance gains of 2–5 dB (as shown in Table III) over standard techniques such as pseudo-Gray coding, the cost incurred is the large variation in the signal energy levels for transmission of different bits. The peak to average ratio of transmission energy for the case of a VQ designed for a Gauss–Markov source is shown in Table I. In this example the resulting PAR is over 2 dB, thereby increasing the linearity requirements of the power amplifier used to transmit the modulated signal.

III. TEA AND QAM TRANSMISSION

The application of TEA, as described in Section II, enhances robustness but also increases the resulting PAR substantially. However, this conclusion is based on treating the modulation procedure as one dimensional. In practice, we are mostly interested in quadrature amplitude modulation (QAM) where, we transmit information on the in-phase and quadrature phase components of a carrier. In the case of binary modulation, the resulting two-dimensional signals are in the form of 4-QAM. Here, we show that by appropriately grouping the bits, the PAR requirements on the two-dimensional QAM signals can be drastically reduced. In fact we show that the PAR can be made equal to 0 dB, with negligible loss in robustness.

Let the n bits be arranged in decreasing order of sensitivities. If the bits i and j are transmitted on a single modulation interval, the two-dimensional signal transmitted is $(b_i\sqrt{e_i}, b_j\sqrt{e_j})$, where $b_i, b_j = \pm 1$. The corresponding signal energy is $e_i + e_j$. Consider the following grouping of bits: $(1,n),(2,n-1),\cdots,(\frac{n}{2},\frac{n}{2}+1)$, where n is assumed to be even[1]. To illustrate the effect of such grouping, we list in Table II the energy of the signals after grouping of bits for our VQ example. We see that the rearrangement drastically reduces the PAR requirements to a fraction of a decibel compared to the large values listed in Table I.

Next, we consider the problem of optimizing TEA subject to the constraint that all the two-dimensional signals have equal energy. This extends the application of TEA to constant envelope modulation methods such as 4-PSK. We achieve this objective by imposing the following explicit constraint: $e_1 + e_n = e_2 + e_{n-1} = \cdots =$ constant. We will refer to this method as constrained TEA. The constraint ensures that the resulting PAR is 0 dB—the same as when we transmit all the bits with equal transmission energy. Such a requirement may, of course, compromise robustness to channel noise. Table III demonstrates the effect of constrained TEA on the robust performance of a VQ. The optimization of TEA is performed for assumed channel SNR of 8 dB, while the performance is evaluated for channel SNR in the range of 4–10 dB. We see that over 95% of the gains (compared to the conventional pseudo-Gray coding [7]) of the optimal TEA, which range from 2 to 5 dB, are captured by constrained TEA. In other words, we have shown that essentially all the advantages of TEA can be retained, while maintaining perfect PAR of 0 dB, as required by 4-PSK modulation.

IV. CONCLUSION

In this letter we addressed the practical application of TEA within the context ofthe commonly used QAM and

[1] If n is odd, we consider two consecutive codevectors and group the bits as $(1,n),(1',n'),\cdots,(\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil')$, where i and i' denote the i th bit of the two codevectors.

constant envelope modulation schemes. The focus was on the application of TEA to a VQ-based communication system, but the ideas presented have wider applicability. We showed that by rearranging the VQ bits appropriately, and subsequently transmitting them as two-dimensional QAM signals, the PAR requirements due to application of TEA can be substantially reduced. We also demonstrated that constrained TEA captures most to all of the gain of optimal TEA while maintaining a perfect PAR of 0 dB, thereby extending the application of TEA to constant envelope modulation.

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