Vector Quantization with Transmission Energy Allocation for Time-Varying Channels

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***Abstract*—This work is concerned with the problem of designing robust, vector quantizer (VQ)-based communication systems for operation over time-varying Gaussian channels. Transmission energy allocation to VQ codeword bits, according to their error sensitivities, is a powerful tool for improving robustness to channel noise. The power of this technique can be further enhanced by appropriately combining it with index assignment methods. We pose the corresponding joint optimization problem and suggest a simple iterative algorithm for finding a locally optimal solution. The susceptibility of the solution to poor local minima is significantly reduced by an enhanced version of the algorithm which invokes the method of noisy channel relaxation whereby the VQ system is optimized while gradually decreasing the assumed level of channel noise. In a series of experiments, the resulting combined technique is shown to outperform standard pseudo-Gray coding by up to 3.5 dB and to exhibit graceful degradation at mismatched channel conditions. Finally, we extend these ideas to the case where both the transmitter and the receiver have information on the current state of a time-varying channel. The proposed method is based on switched encoding and adaptive decoding. Experimental results show that the proposed system achieves close to optimal performance.**

***Index Terms*— Index assignment, modulation, source-channel coding, vector quantization.**

I. Introduction

With the advent of wireless personal communication systems, there has been a growing interest in the area of robust source coder design. To see the motivation more clearly, let us consider an example of a communication system which transmits a coded source over a mobile, cellular, time-division multiple access (TDMA) channel employing binary modulation (either BPSK or QPSK). It is known that mobile communication channels are characterized by a time-varying multipath delay spread [23]. In the case of TDMA, this results in intersymbol interference which requires the use of an equalizer at the receiving end. The signal-to-noise (and interference) ratio at the output of the equalizer determines the decoded bit error rate. Let the system design be such that, most of the time, the SNR at the output of the equalizer is high enough to assure a low decoded bit error rate. However, occasionally, due to poor multipath delay spreads, the SNR level at the equalizer output will drop significantly, resulting in high decoded bit error rate and poor overall performance. One natural approach to combat the effect of time-varying channel conditions is to employ an appropriate error correcting code and increase the level of protection provided to the source coder bit stream. Error correcting codes cost in bit rate; that is, a fraction of the available source bits must be sacrificed for channel coding. Thus, the performance of the system under poor channel conditions is improved at the cost of reduced performance under clean (or good) channel conditions. An alternative approach replaces separate channel coding with a robust source coder that utilizes all the available bit rate. The objective here is to design the source coder with an in-built robustness to channel variations. With this approach it is possible to avoid (or, in the case of channel optimized quantization, reduce) the compromise in overall system performance under clean channel conditions (which predominate most of the time) while improving the system robustness to occasional poor channel conditions.

In this work we pursue the latter approach and focus on the design of robust source coders. In particular, we consider the design of robust vector quantizers (VQ). Existing methods for robust VQ-based communications fall into two broad categories: index assignment and channel-optimized quantization. Index assignment (IA) techniques [3], [31] employ a source-optimized VQ. (By source-optimized VQ we mean a VQ system optimized under the assumption of error-free transmission.) This ensures that the system performance is uncompromised when the channel is clean. Robustness to channel errors is achieved by judicious assignment of indices to codevectors, so that the damage caused by the most frequent decoder errors is minimized. The optimization of IA is normally performed for some representative (typical or average) channel condition. However, the resulting IA is known to achieve robust performance over a large range of channel conditions. Channel-optimized quantization [4], [19] assumes precise knowledge of the channel characteristics. This knowledge is used to modify the VQ codebook and the encoding rule so as to achieve optimal performance under the prescribed channel conditions. This class of methods is more

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channel specific and achieves better performance compared to index assignment techniques at the prescribed channel conditions. However, channel optimized quantization incurs some performance degradation under clean channel conditions.

In this paper we explore the potential advantages of optimizing the modulation scheme to increase the robustness of VQ-based communication systems. This work has its roots in the early work of Bedrosian [1] and the subsequent work of Sundberg [27] (see also [30]), where the idea of unequal allocation of transmission energy to the various bits was considered in the context of pulse-coded modulation (weighted PCM). Recently, it was demonstrated in [7] and [8] that the performance of a VQ over a noisy channel can be substantially improved by allocating transmission energy to the VQ output bits according to their sensitivity. In the area of multicarrier modulation-based communication, similar results were independently obtained by Ho and Kahn [13], [14]. The basic approach for transmission energy allocation (TEA) for VQ communications was described in [7] and [13]. Later work reported on extensions to the case of multistage VQ [8] and channel optimized VQ [14].

While TEA for robust vector quantization is a direct generalization of the early weighted PCM, it is also related to the more recent work on optimal design of multidimensional constellations for joint source-channel coding [21], [28]. These papers consider the joint design of a VQ and a multidimensional constellation, under an average power constraint. The problem is very general (and very hard to solve), and it may be viewed as including TEA as an important, nontrivial, special case. Specifically, when the optimization of the multidimensional constellations is performed under the constraint that the resulting constellations be effectively binary in each modulation interval, then the solution would be a TEA solution. Imposing this structural constraint on the constellations drastically simplifies the problem of constellation design which becomes practical. Moreover, implementing binary modulation can be significantly simpler than general multidimensional constellations. Hence, imposition of such a structure may often be desirable in practice.[1]

This paper is organized as follows: In Section II we briefly review the basic idea of TEA in the context of a VQ indexed by the natural binary code (NBC). Natural binary code is a codebook indexing obtained from VQ design using the splitting initialization, see, e.g., [4]. NBC is a convenient indexing for VQ codewords and its combination with TEA provides substantial improvements over standard pseudo-Gray index assignment. However, NBC is not the optimal choice. In Section III we pose the problem of joint optimization of TEA and index assignment. We develop a direct optimization algorithm for this problem and show that considerable improvements over TEA-NBC can be achieved. The direct optimization method is, nevertheless, susceptible to poor local minima that riddle the cost surface due to the complex discrete nature of the index assignment problem. To overcome this difficulty, an enhanced version of the algorithm is proposed which incorporates noisy channel relaxation and has the ability to avoid many shallow local minima. Finally, in Section IV we extend these ideas to the case where the characteristics of the time-varying channel are known. Here we focus on adaptation of energy allocation and VQ design to the current state of the channel and describe a method based on switched encoders and adaptive decoding.

[1] Even in the context of data transmission, there exists literature on constellation optimization, e.g., [5]. However, constellations with more structure are usually preferred in practice.

II. Transmission Energy Allocation

In this section we briefly review the basic TEA-NBC method [7], [14], where transmission energy allocation is applied in conjunction with the natural binary code. We also include extensions of [7] to handle the case of an arbitrary VQ (which is not necessarily source-optimized) and comment on the choice of the representative channel conditions.

Consider a source that produces a sequence of independent random vectors x, and the corresponding source-optimized VQ with its codebook $C = \{y_0, y_1, \cdots, y_{2^n-1}\}$. Given x, the VQ encoder finds the nearest codevector y_I and employs binary modulation to transmit the n-bit index I over a memoryless Gaussian channel whose SNR varies with time.[2] The decoder receives a noisy version of the transmitted signal and applies hard decision decoding to obtain the received index J. Note that we could equivalently state that individual bits are transmitted on independent binary symmetric channels, whose bit error rates depend on the corresponding Gaussian channel SNR. Given the received index J, the decoder produces the codevector y_J as an estimate of the source vector x. Since the VQ is source-optimized, its codevectors satisfy the centroid rule [20]. This implies that the overall distortion can be decomposed into quantization and channel distortion terms

$$D = E||x - y_J||^2 = \overbrace{E||x - y_I||^2}^{D_q} + \overbrace{E||y_J - y_I||^2}^{D_c}. \quad (1)$$

Index assignment aims at minimizing D_c by a judicious assignment of binary indexes to the codevectors (see, e.g., [3] and [31]). As D_c depends on the channel conditions, IA typically assumes a particular representative (or expected) level of channel noise for the design phase. It is well known that some standard VQ design methods naturally produce relatively good indexing. An important example is the NBC which is obtained from VQ design initialized by the splitting method.

Of particular relevance to us is the fact that, in general, the index bits are not equally sensitive to channel errors. To formally define the error sensitivity of bits, let us employ a bitwise explicit notation for the transmitted index: $I = (i_1 i_2 \cdots i_n)$. The sensitivity of the jth bit is defined as the expected amount of distortion caused by a bit error at this

[2] As mentioned in Section I, in TDMA mobile communication we encounter a situation where the value of the SNR at the equalizer output varies with the time-varying channel conditions. If we assume that equalization cancels all intersymbol interference and model the overall noise at the output as white Gaussian, the resulting channel becomes a time-varying memoryless Gaussian channel. This channel model also applies to mobile communication scenarios characterized by flat Rayleigh fading [23].

location

$$D_j = E||y_{i_1, i_2, \cdot, i_j, \cdot, i_n} - y_{i_1, i_2, \cdot, i_j^*, \cdot, i_n}||^2 \tag{2}$$

where "superscript *" denotes the complement: $i^* = 1 - i$. In [7] it was demonstrated that there is large variation in sensitivity among the bits of a typical VQ indexed by NBC. This variation in bit sensitivity is exploited by providing optimal, unequal error protection via the allocation of transmission energy to the various VQ bits.

The n index bits are transmitted independently on a time-varying Gaussian channel using binary modulation. In practice, much of the variation in the channel SNR is due to variation in the power level of the received signal. We find it convenient to consider instead the equivalent formulation where the received signal power remains constant while the level of channel noise varies. During the design phase, we optimize the system for a selected representative channel condition. Let σ_r^2 be the level of the representative Gaussian channel noise. (See the remark at the end of this section for a discussion on the choice of σ_r^2.) Let us denote by e_j the energy allocated to the jth bit, hence, the corresponding bit error rate is

$$\epsilon_j = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{e_j/\sigma_r^2}}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt.$$

If, for the sake of computational simplicity, we neglect the probability of more than one bit error in an index transmission, the distortion due to channel errors simplifies to

$$D_c = \sum_{j=1}^{n} D_j \epsilon_j. \tag{3}$$

The general case where we consider the effect of more than one bit error in index transmission is addressed via (5). We wish to minimize D_c over all choices of $\{e_j\}$, that is, by allocating transmission energy to the n bits, subject to the constraint on the total energy available for their transmission

$$\sum_{j=1}^{n} e_j = e_{\text{tot}}.$$

This constrained optimization problem can be solved using various techniques. A reasonable approach is to evaluate the set of derivatives $\{\partial D_c/\partial e_j\}$ and use them in either a gradient descent algorithm or a greedy energy quanta allocation algorithm, similar to the one described in [7].

1) Note: Evaluation of $\partial D_c/\partial e_j$ using (3) is based on two practically limiting assumptions, namely, single bit errors and the centroid rule. The assumption of single bit errors was made in order to simplify the evaluation of $\{\partial D_c/\partial e_j\}$. However, if the representative level of channel noise is sufficiently high, the probability of multiple bit errors per index will no longer be negligible. Also, in a more general setup where we do not necessarily have a source-optimized VQ, the codevectors may not satisfy the centroid rule. For these cases, we proceed as follows to evaluate $\partial D_c/\partial e_j$: The overall distortion is given more generally by

$$D = E||x - y_J||^2 = E||x - c_I||^2 + \overbrace{E||y_J - c_I||^2}^{D_c} \tag{4}$$

where c_I is the centroid of the encoding region indexed by I. $\partial D_c/\partial e_j$ is now evaluated directly as

$$\frac{\partial D_c}{\partial e_j} = \sum_I P(I) \frac{\partial P(J|I, \sigma_r^2)}{\partial e_j} ||y_J - c_I||^2 \tag{5}$$

where $P(I)$ is the *a priori* probability that index I is transmitted (estimated from the training set), and $P(J|I, \sigma_r^2)$ is the probability of decoding index J given transmission of I and channel noise level of σ_r^2.

2) Performance of TEA-NBC: Table I provides a comparison of the performance of TEA-NBC with that of standard pseudo-Gray (PG) coding. (The table also includes other results that should be ignored for the moment.) The PG method consists of index assignment only and allocates equal transmission energy to all the bits. The results are given for a first-order Gauss–Markov source. TEA-NBC and PG are optimized for a representative average channel SNR of 8 dB. The performance of the system is tested as the value of the average channel SNR varies from 4 to 10 dB (such channel variations are typical in the case of fading). All the values given in Table I and elsewhere in this paper depict the performance evaluated over test sets. The results show that in many cases TEA-NBC can achieve large performance gains. The gains are particularly important under conditions of heavy

TABLE I
OVERALL PERFORMANCE COMPARISONS OF: (PG)—PSEUDO-GRAY CODING WITH EQUAL ENERGY ALLOCATED TO ALL THE BITS; (TEA-NBC)—NATURAL BINARY CODE FOLLOWED BY ENERGY ALLOCATION; (TEA-IA)—ITERATIVE OPTIMIZATION OF INDEX ASSIGMENT AND ENERGY ALLOCATION; (TEA-NCR)—ITERATIVE APPLICATION OF NOISY CHANNEL RELAXATION AND ENERGY ALLOCATION. THE SYSTEM PERFORMANCE IS MEASURED IN TERMS OF THE OVERALL SNR IN DECIBELS. THE SOURCE IS GAUSS–MARKOV WITH CORRELATION COEFFICIENT ρ. THE VQ CODEBOOK SIZE IS 256. THE DESIGN WAS OPTIMIZED FOR CHANNEL SNR OF 8 dB, AND THE PERFORMANCE WAS EVALUATED ON A TEST SET FOR THE CHANNEL SNR RANGE OF 4–10 dB

Channel SNR		dim = 2			dim = 4		
		$\rho = 0.0$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.0$	$\rho = 0.8$	$\rho = 0.9$
4 dB	PG	4.43	4.63	5.03	3.60	4.52	4.75
	TEA-NBC	5.09	6.40	7.14	2.72	5.01	5.84
	TEA-IA	5.92	6.49	7.16	3.68	5.29	6.29
	TEA-NCR	6.39	7.51	7.79	4.31	6.13	6.64
6 dB	PG	8.12	8.40	8.86	6.11	7.60	8.08
	TEA-NBC	9.36	10.94	11.93	5.51	8.30	9.57
	TEA-IA	10.03	11.05	11.96	6.17	8.39	9.82
	TEA-NCR	10.48	11.88	12.38	6.72	9.09	10.10
8 dB	PG	13.35	13.86	14.43	8.55	11.03	12.15
	TEA-NBC	14.78	16.58	17.78	8.30	11.50	13.22
	TEA-IA	15.27	16.67	17.78	8.60	11.52	13.35
	TEA-NCR	15.56	17.23	18.01	8.87	11.84	13.47
10 dB	PG	18.79	20.18	21.21	9.71	13.03	14.95
	TEA-NBC	19.11	20.97	22.46	9.68	13.07	15.03
	TEA-IA	19.31	20.97	22.46	9.74	13.10	15.16
	TEA-NCR	19.38	21.29	22.35	9.83	13.12	15.16

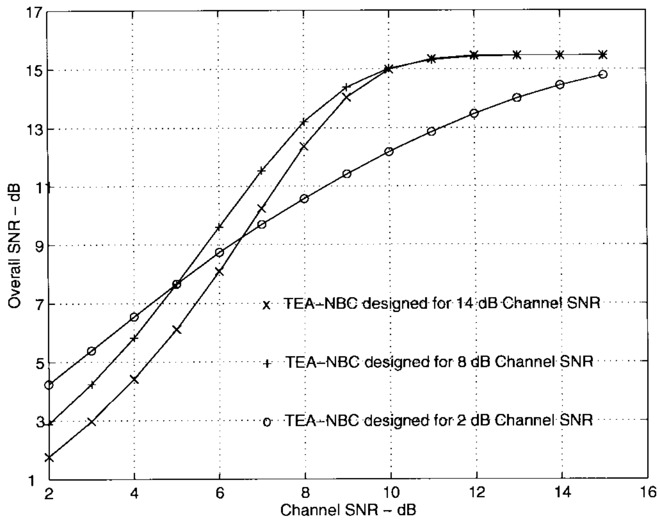

Fig. 1. Effect of the representative channel conditions σ_r^2 on TEA-NBC performance under channel mismatch conditions. The source is Gauss–Markov with correlation coefficient $\rho = 0.9$, the vector dimension is four, and the codebook size is 256.

channel noise (resulting from severe fading), where, for some cases the performance gains can be of the order of 2 to 3 dB. Thus, exploiting variation in bit sensitivities via TEA is a promising direction.

3) Remark: The optimization of the transmission energy allocation is performed assuming a representative noise level σ_r^2. The choice of σ_r^2 can have a significant impact on the resulting robustness of the VQ. If the value chosen for σ_r^2 is too small, almost uniform protection is provided to the bits. Hence, we fail to take advantage of the varying bit sensitivities. An excessively high value for σ_r^2 results in allocation of very little or no transmission energy to the least sensitive bits, thereby causing a substantial performance loss when the channel is cleaner. In Fig. 1 we demonstrate the effect of the representative channel condition σ_r^2 on TEA-NBC performance. It is evident that the system optimized for channel SNR of 14 dB suffers in performance under conditions of heavy channel noise. On the other hand, the system designed for 2 dB performs very poorly when the channel noise is low, due to allocation of very small amount of energy to the less sensitive bits. However, a sensible choice of σ_r^2 should reflect the channel conditions that we expect to encounter on the average. If the expected channel conditions translate into excessive level of σ_r^2, it suggests that the amount of available transmission energy is too low for meaningful transmission of all VQ index bits. Under such circumstances it might be worthwhile to either increase the total transmission energy (if possible), or reduce the number of bits used by the VQ.

III. Joint Optimization of Index Assignment and Transmission Energy Allocation

While substantiating the promise of TEA, the results of Table I also indicate that the choice of NBC, though a natural starting point for applying TEA, cannot guarantee success. It can be seen that the gains of TEA-NBC diminish with increase in VQ dimension and with decrease in the correlation coefficient. For example, with $\rho = 0.0$ and dim $= 4$, PG happens to outperform the TEA-NBC scheme. These observations motivate the search for better methods for combining index assignment with TEA. Another drawback of TEA-NBC lies in the fact that NBC dictates the use of the splitting VQ design method which is itself suboptimal. It is often possible to design better source-optimized VQ by adopting more elaborate techniques such as deterministic annealing [24] or stochastic relaxation [32]. It is clearly desirable to have a method that exploits the advantages of TEA and yet is generally applicable to any given VQ. This gives further motivation for joint optimization of index assignment and energy allocation.

We first propose a simple and direct approach for joint optimization of IA and TEA. It is based on iterative application of the IA and TEA procedures. However, this technique is susceptible to poor local minimum traps. To attack this problem, we then propose a second, more involved technique which incorporates the idea of noisy channel relaxation.

A. Locally Optimal Design

An optimal solution to the problem of joint design of IA and TEA should satisfy the following straightforward conditions.

Condition 1: Transmission energy allocated to the different bits must be matched to the bit-sensitivities resulting from the underlying IA.

Condition 2: Indexing of the codevectors must be optimal for the underlying transmission energy allocation.

Given any IA, condition 1 can be satisfied by evaluating the bit sensitivities (due to this indexing) followed by optimization of the transmission energy allocation. Condition 2 is satisfied by optimizing the IA for the given energy allocation. In this context we note that many IA algorithms, such as binary switching [31], can be easily modified to take into account unequal bit error rates for the various bits. Hence, such a modified IA technique can be used to optimize the index assignment for a given transmission energy allocation. These observations motivate the following simple strategy for joint optimization of TEA and IA.

1) Start with an initial index assignment.
2) Evaluate the sensitivities of the various bits and apply the energy allocation algorithm.
3) Reassign indexes to the codevectors via an IA technique (e.g., binary switching) which has been modified to include the effect of unequal bit error rates.
4) Check convergence (e.g., $\Delta D_c/D_c$ below threshold), if so, stop.
5) Go to step 2.

The algorithm alternates between the TEA and IA steps. Since D_c is monotonically decreasing with each step, we are ensured of obtaining a locally optimal solution to the problem. This algorithm will be referred to as TEA-IA.

We applied TEA-IA to a VQ that was designed for the first-order Gauss–Markov source of Section II. It was found that the solution depends heavily on the initial index assignment. We experimented with the following two initializations and chose the one that gave the best performance in each case: a) NBC and b) NBC followed by pseudo-Gray coding. These results were added to Table I. The binary switching algorithm [31] was modified to account for the variation in error rates among the bits and used for optimization of IA. The design assumed channel SNR of 8 dB, while the performance was evaluated at the channel SNR range of 4–10 dB. The iterative optimization of TEA-IA achieves a rearrangement of the code vectors that yields additional modest improvements in performance (up to 0.8 dB). The performance improvements are more pronounced under conditions of heavy channel noise.

It is important to note that larger improvements were achieved wherever TEA-NBC provided little or no gains over the pseudo-Gray method. This observation indicates the importance of joint optimization of IA and TEA.

B. Noisy Channel Relaxation

The significant impact of initialization on the quality of the solution to the joint optimization problem is not surprising and stems from the well documented local minima problem of IA methods. Here, the susceptibility of the solution to poor local minima is exacerbated with the increased complexity of the joint IA and TEA optimization problem. In fact, our experiments show that some initializations resulted in extremely poor solutions. To overcome this shortcoming, we developed a method that has an enhanced capability to avoid these poor local minima.

The problem of poor initialization for IA can be tackled effectively via the noisy channel relaxation (NCR) technique. A suggestion to vary the level of channel noise during VQ design appeared first in [16] in the context of sequential design with self-organizing feature maps [17]. It was demonstrated in [16] that by letting the level of channel noise approach the prescribed value in a somewhat oscillating manner, poor local minima solutions in the channel optimized VQ design may be avoided. Independently, in [6] the noisy channel relaxation technique was developed for generalized Lloyd algorithm (GLA), the standard batch method for VQ design. The basic idea of NCR is as follows: Instead of designing a source optimized VQ, we begin by designing a channel-optimized VQ for a high level of channel noise and then gradually reduce the level of channel noise assumed for the design. Channel optimized VQ design for a given level of channel noise is performed using the noisy channel generalized Lloyd algorithm (NC-GLA), see, e.g., [4]. This gradual reduction of design noise level, or noisy channel relaxation, provides means for avoiding many poor local minima of the IA problem. Initializing the iterations at a very high level of channel noise makes it easier for the system to find a good initial IA which is then tracked and reoptimized as the noise level is reduced. The final iterations are performed for a noiseless channel, thereby yielding a source-optimized VQ, albeit with a built-in indexing inherited from the design in earlier stages. NCR has been demonstrated to substantially improve the robustness of the VQ to channel errors.

In this work we extend NCR for application to the problem of joint IA-TEA optimization. We start with the observation that NC-GLA can also be modified in a straightforward manner to account for variation in error rates among bits. This modified NC-GLA can therefore be used to incorporate NCR within the joint optimization procedure. To design a VQ via NCR for a given energy allocation, we proceed as follows. We use a very high variance of Gaussian channel noise in the initial iterations and reduce the noise variance as the iterations proceed. At each iteration we use the current level of channel noise to evaluate the bit error rates for the different VQ output bits. Using these values in the NC-GLA method, we perform the channel-matched VQ design. The iterations toward the end are performed with zero noise, yielding a "noiseless channel"-optimized (i.e., source-optimized) VQ with indexing that takes into account the transmission energy allocated to the various bits.

We now summarize the TEA-NCR algorithm, which integrates index assignment using noisy channel relaxation and energy allocation, as follows.

1) Initialize the transmission energy allocated to the various bits.
2) Design the VQ using NCR. (The final index assignment may be fine-tuned by the binary switching algorithm.)
3) Evaluate the sensitivities of the various bits and reoptimize the transmission energies allocated to the VQ bits.
4) Check for convergence, e.g., if $\Delta D/D < Th$ stop.
5) Go to step 2.

The overall performance obtained by the TEA-NCR approach is tabulated in Table I. We can see that TEA-NCR is able to avoid many of the poor local minima that trap TEA-IA yielding improvements of up to 0.8 dB. The overall improvement in performance over standard pseudo-Gray coding is in the rough range of 0.6–3.5 dB under heavy channel noise, while the net improvement over TEA-NBC is of the order of 0.6–1.3 dB.

Details of the Relaxation Schedule: The iterative VQ design was performed using NC-GLA starting with noise level corresponding to a channel SNR of -5 dB. Under these conditions, the number of nonempty encoding regions [6], [12] resulting from channel optimized VQ design is much smaller than the target size of the VQ. The variance of noise was reduced by a factor of 1.01 in each NC-GLA iteration. As the variance of the channel noise is reduced, the number of encoding regions increases. The rate of increase of the number of nonempty encoding regions was found to be an effective means of regulating the relaxation schedule. If the number of nonempty encoding regions increased by more than two in a single iteration, the reduction of noise level was stopped. The relaxation was subsequently resumed when the number of nonempty encoding regions remained the same over two consecutive iterations. When the number of nonempty encoding regions becomes equal to the target size of the VQ, the noise level reduction schedule was accelerated to a factor of 1.1 per iteration.

IV. Extension to the Informed Receiver/Informed Transmitter Case

We now extend the idea of transmission energy allocation to the case of channel optimized quantization. Unlike the index assignment techniques, here we assume more precise information about the channel conditions. In many cases of practical interest, such as mobile communication systems, it is possible to estimate the state of the channel at the receiver. Depending on the feasibility of a feedback path, the transmitter may or may not have knowledge of the state of the channel. Communication systems can, hence, be classified [15] into the following two conditions: i) informed transmitter and informed receiver—where a feedback path from the receiver to the transmitter exists; ii) informed receiver and uninformed transmitter—no feedback path.

In this section we address the design of a VQ with TEA for the informed receiver/informed transmitter case. We first summarize the design of a channel-optimized VQ with TEA (COVQ-TEA). Employing a VQ optimized for the current level of channel noise is an optimal solution when both the transmitter and the receiver know the state of the channel. The drawback of this solution is due to the fact that this system requires the availability of optimized VQ's for all possible channel conditions. The implied storage complexity is impractical. However, the superior performance of the COVQ-TEA over an index assignment system provides a strong motivation to investigate methods that approximate COVQ-TEA while maintaining feasible complexity. The remainder of the paper is devoted to this objective.

A. Channel-Optimized VQ with TEA

In this subsection we briefly summarize the design of VQ with TEA matched to a particular level of channel noise σ_n^2. This algorithm will be referred to here as COVQ-TEA. A derivation of a similar algorithm in the context of multicarrier modulation was presented in [14]. A brief summary of the basic design algorithm is followed by a discussion of useful initializations. The superior performance of COVQ-TEA *at the prescribed channel noise level* is demonstrated by comparing it with the performance of TEA-NCR.

First we describe the design of COVQ without consideration of energy allocation. The COVQ encoding rule is specified by the parameters $\{(u_I, \theta_I)\}$ and determines the index I to be transmitted as follows (see, e.g. [6] and [12]). Let x be the input vector to be quantized, we transmit the index I if

$$||x - u_I||^2 + {\theta_I}^2 \leq ||x - u_{I'}||^2 + \theta_{I'}^2 \qquad \text{for all } I' \tag{6}$$

where u_i and θ_i are given below. The decoder is a lookup table which produces an estimate $y_J = E[x|J, \sigma_n^2]$ on receiving index J. The encoding rule is related to the decoder codebook $\{y_J\}$ via

$$u_I = \sum_J P(J|I, \sigma_n^2) y_J \tag{7}$$

and

$${\theta_I}^2 = \sum_J P(J|I, \sigma_n^2) ||y_J||^2 - ||u_I||^2. \tag{8}$$

TABLE II

Comparison of the Performance of COVQ-TEA Designated via Noisy Channel Relaxation and Natural Binary Code Initialization. The Source is Gauss–Markov with Correlation Coefficient ρ, the Vector Dimension is Four, and the Codebook Size is 256. The Performance on a Test Set is Measured in Terms of Overall SNR in Decibels

Channel SNR (dB)		3	4	5	6	7	8
$\rho = 0.0$	NCR Design	4.67	5.51	6.41	7.34	8.24	9.01
	NBC Initialization	4.49	5.21	5.90	6.63	7.55	8.52
$\rho = 0.8$	NCR Design	6.84	7.79	8.83	9.94	11.07	12.08
	NBC Initialization	6.71	7.66	8.67	9.77	10.74	11.77

Hence, the COVQ system is specified completely by the encoding rule $\{(u_I, \theta_I)\}$ and the decoder codebook $\{y_J\}$. The design of a channel-optimized VQ is based on a modified version of the standard GLA [20] that takes the effects of channel noise into account. This modified version is referred to as noisy channel GLA (NC-GLA) [4], [19].

Although the NC-GLA algorithm has been described for the case of equal bit error rates for the VQ bits, it can be extended in a straightforward manner to account for the unequal bit error rates. The COVQ-TEA algorithm consists of alternating two steps: given energy allocation redesign VQ by the modified NC-GLA; given the new VQ, reoptimize energy allocation by TEA wherein we evaluate $\partial D_c / \partial e_j$ as described in (5). The COVQ-TEA is summarized below.

1) Choose an initial energy allocation and VQ design.
2) Redesign the VQ using NC-GLA.
3) Reoptimize the transmission energy allocated to the VQ bits.
4) If the decrease in overall distortion is below a prespecified threshold, stop.
5) Go to step 2.

The overall distortion is monotone decreasing and convergence to a local minimum is ensured.

1) Design of COVQ-TEA via Noisy Channel Relaxation: The design of COVQ-TEA is based on a local descent iterative algorithm that converges to a local optimum. The final solution that we obtain is, however, influenced by the initialization. It was suggested in [14] that a good initialization for this algorithm is to start with the VQ indexed by a natural binary code and the corresponding energy allocation. In our experiments we found that better solutions can usually be obtained by employing noisy channel relaxation. The basic idea is to choose a sequence of channel noise levels of decreasing variance—$\sigma_0^2 > \sigma_1^2 > \cdots > \sigma_n^2$. Where the final value σ_n^2 denotes the prescribed (target) variance of channel noise. We start by designing COVQ-TEA for a channel with noise variance σ_0^2. The resulting VQ and energy allocation is used as the initialization for COVQ-TEA design at noise variance σ_1^2. We continue in this manner until we have designed a VQ optimized for noise variance σ_n^2. The improvement in performance due to NCR is tabulated in Table II. The channel noise level σ_0^2 was chosen to correspond to a channel SNR of -5 dB. The relaxation sequence consisted of channel SNR steps of 1 dB. Improvements of up to 0.5 dB over the standard NBC initialization were achieved.

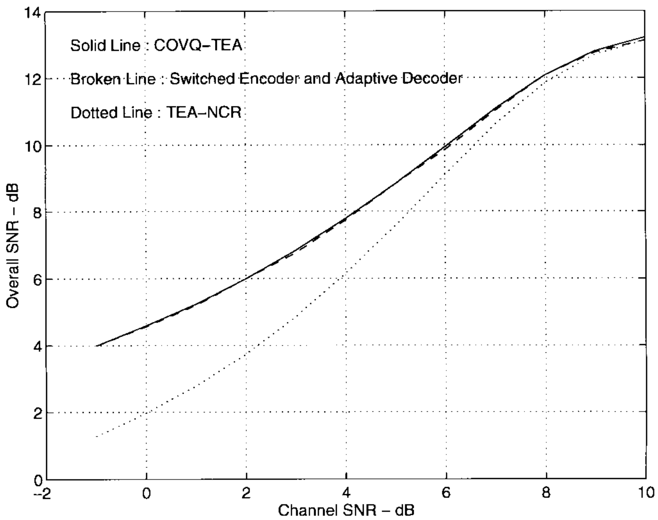

Fig. 2. A performance comparison of COVQ-TEA with TEA-NCR. The performance of the switched-encoder-based adaptive system is also shown. The source is Gauss–Markov with correlation coefficient $\rho = 0.8$, the vector dimension is four, and the codebook size is 256.

2) Performance Comparison for the Informed Receiver Informed Transmitter Case: Fig. 2 shows the performance advantage of COVQ-TEA over an index assignment system which employs a source-optimized VQ. The source is Gauss–Markov with auto-correlation coefficient 0.8, which is blocked into 4-tuples. The results show substantial improvements of COVQ-TEA over the index assignment scheme. Moreover, the improved performance is most pronounced under conditions of heavy channel noise. The fact that the encoder–decoder pair employed by TEA-NCR is optimized for a noiseless channel explains the narrowing gap in performance under conditions of low channel noise.

B. Switched–Encoder COVQ System for a Time-Varying Channel

The results of the last subsection demonstrate that when both the transmitter and receiver have information about the true channel conditions, COVQ-TEA matched to the current level of channel noise provides excellent performance. However, if we wanted to employ such a system we would need to store the VQ encoder–decoder pairs for an impractically large number of possible channel conditions. What we desire is a system with an ability to adapt itself to time-varying channel conditions while maintaining manageable storage and computational complexity. Toward this end, we investigated the performance of systems consisting of switched encoders and adaptive decoders. Note that the impractical system of COVQ matched to the exact channel conditions serves as a useful upper bound on the performance achievable by the switched encoder system.

1) Switched Encoder: We design and store the encoding rules for a small number of selected channel conditions. The transmitter uses the information about the current level of channel noise along with a switching strategy to select one of the available encoding rules.

2) Adaptive Decoder: The decoder knows the current level of channel noise, σ_n^2. On receiving the index J, it produces an estimate y_J to minimize $E\big[||x-y||^2|J, \sigma_n^2\big]$. The corresponding estimate y_J is given by

$$y_J(\sigma_n^2) = E\big[x|J, \sigma_n^2\big] = \frac{\sum_I P(I)P(J|I, \sigma_n^2)c_I}{\sum_I P(I)P(J|I, \sigma_n^2)} \tag{9}$$

where the quantities $\{P(I)\}$ and $\{c_I\}$ denote the probability that index I is transmitted and the centroid of the encoding region indexed by I, respectively.

We make the following observations on the implementation and feasibility of a switched encoder/adaptive decoder system.

1) Implementation of the decoder based on (9) requires storage of the quantities $\{P(I), c_I\}$. Depending on the current level of channel noise, the decoder selects the set $\{P(I), c_I\}$ corresponding to the encoding rule used by the transmitter.
2) The quantity $P(J|I, \sigma_n^2)$ can be estimated at the decoder since it depends on the current level of channel noise and energy allocation at the encoder, both of which are known to the decoder.
3) The complexity involved in computing y_J is proportional to the size of the codebook. In other words, the decoding complexity is of the same order as the encoding complexity. Hence, we may safely assume that *a decoder which computes $y_J(\sigma_n^2)$ using* (9) is practically feasible whenever the corresponding encoder is.

The results obtained by the switched–encoder/adaptive decoder system for the case of Gauss–Markov source are included in Fig. 2. The vector dimension used is four and the correlation coefficient is 0.8. The encoder chooses from a set of four encoding rules corresponding to channel SNR of -1, 2, 5, and 8 dB. The switching strategy is based on a lookup table that contains a list of the best encoding rules to be used for given channel conditions. It is evident that the combination of switched encoder and adaptive decoder achieves performance close to the ideal upperbound obtained by COVQ-TEA, although only four encoding rules are available at the transmitter. Thus, the system is capable of adapting itself to the characteristics of a time-varying channel while maintaining manageable storage and computational complexity.

Remarks:

1) We have not tried to optimize the choice of the values of channel SNR for which the encoding rules employed by the switched encoder are designed. However, since the overall performance is so close to the upperbound, such optimization cannot produce any significant improvement in performance. One may also consider adaptation

of the encoder to the channel conditions on a continuous scale along the lines of the discussion in [12].

2) Other approaches, where the decoder is adapted to the current level of channel noise, are described in literature. The fact that an optimal estimate of the source vector can be obtained by direct computation as described in (9) was observed in [12], [21], [25], and [26]. A decoding strategy based on linear mapping of block codes is described in [26], while [18] investigates the use of multiresolution codebooks. An extensive description of several simplifications of the optimal adaptive decoder are described in [12]. Based on the arguments presented herein, however, we emphasize that the complexity of the *optimal* decoder is manageable, and hence simpler but suboptimal decoding schemes may often be unnecessary.

V. Conclusions and Discussion

This work is concerned with the design of a robust, VQ-based communication system for operation over a time-varying Gaussian channel. Motivated by the early work of Bedrosian, we investigated the idea of transmission energy allocation to provide unequal error protection to the various bits of a vector quantizer according to their importance. We demonstrated significant performance gains resulting from an appropriate combination of TEA and index assignment techniques. The problem of joint index assignment and TEA is, however, susceptible to poor local minima. We proposed a design method based on noisy channel relaxation that has the ability to avoid many poor local minima. When information about the state of the channel is available at the receiver and the transmitter, the overall performance can be further improved by employing modified decoding/encoding strategies. For this scenario, we developed a switched–encoder-based system capable of adapting itself to time-varying characteristics of the channel.

TEA is a technique to achieve unequal error protection to the different bits. There exist more powerful channel coding methods for unequal protection [2], [9], [22], [29], of which TEA may be viewed as a special case. An important advantage of transmission energy allocation over other techniques is in that unequal protection is achieved without investing additional bits and computational complexity. The fact that no additional bits are required makes TEA an attractive alternative over other UEP methods in certain mobile communication systems such as TDMA.

On the other hand, an undesirable feature of TEA is that, in general, it affects the resulting carrier which will have a time-varying amplitude. It is often desirable to minimize or avoid (if we intend to employ class-C amplifiers) variations in the carrier envelope. However, there are ways to mitigate and even eliminate this difficulty as described in our more recent work [10].

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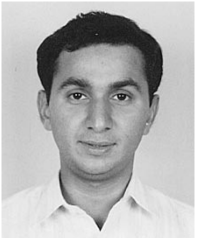

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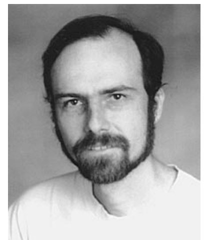

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